MATHEMATICS

A PRACTICAL ODYSSEY

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1.3
TRUTH TABLES

Suppose your friend Maria is a doctor and you know that she is a Democrat. If someone told you "Maria is a doctor and a Republican," you would say that the statement was false. On the other hand, if you were told "Maria is a doctor or a Republican," you would say that the statement was true. Each of these statements is a compound statement—the result of joining individual statements with connective words. When is a compound statement true, and when is it false? To answer these questions, we must examine whether the individual statements are true or false and the manner in which the statements are connected.

The **truth value** of a statement is the classification of the statement as true or false and is denoted by T or F. For example, the truth value of the statement "Santa Fe is the capital of New Mexico" is T. (The statement is true.) In contrast, the truth value of "Memphis is the capital of Tennessee" is F. (The statement is false.)

A convenient way of determining whether a compound statement is true or false is to construct a truth table. A **truth table** is a listing of all possible combinations of the individual statements as true or false, along with the resulting truth value of the compound statement. As we will see, truth tables also allow us to distinguish valid arguments from invalid arguments.

	p
1.	T
2.	F

Figure 1.16

Truth Values for a Statement p

THE NEGATION $\sim p$

The **negation** of a statement is the denial, or opposite, of the statement. (As stated in the previous section, because the truth value of the negation depends on the truth value of the original statement, a negation can be classified as a compound statement.) To construct the truth table for the negation of a statement, we must first examine the original statement. A statement p may be true or false, as shown in Figure 1.16. If the statement p is true, the negation $\sim p$ is false; if p is false, $\sim p$ is true. The truth table for the compound statement $\sim p$ is given in Figure 1.17. Row 1 of the table is read "$\sim p$ is false when p is true." Row 2 is read "$\sim p$ is true when p is false."

	p	$\sim p$
1.	T	F
2.	F	T

Figure 1.17

Truth Table for a Negation $\sim p$

THE CONJUNCTION $p \wedge q$

A **conjunction** is the joining of two statements with the word *and*. The compound statement "Maria is a doctor and a Republican" is a conjunction with the following symbolic representation:

p: Maria is a doctor,

q: Maria is a Republican,

$p \wedge q$: Maria is a doctor and a Republican.

The truth value of a compound statement depends on the truth values of the individual statements that comprise it. How many rows will the truth table for the conjunction $p \wedge q$ contain? Because p has two possible truth values (T or F) and q has two possible truth values (T or F), we need four $(2 \cdot 2)$ rows in order to list all possible combinations of Ts and Fs, as shown in Figure 1.18.

In order for the conjunction $p \wedge q$ to be true, the components p and q must *both* be true; the conjunction is false otherwise. The completed truth table for the conjunction $p \wedge q$ is given in Figure 1.19. The symbols p and q can be replaced by any statements. The table gives the truth value of the statement "p and q," dependent upon the truth values of the individual statements "p" and "q." For instance, row 3 is read "the conjunction $p \wedge q$ is false when p is false and q is true." The other rows are read in a similar manner.

THE DISJUNCTION $p \vee q$

A **disjunction** is the joining of two statements with the word *or.* The compound statement "Maria is a doctor or a Republican" is a disjunction (the *inclusive* or) with the following symbolic representation:

p: Maria is a doctor,
q: Maria is a Republican,
$p \vee q$: Maria is a doctor or a Republican.

Even though your friend Maria the doctor is not a Republican, the disjunction "Maria is a doctor or a Republican" is true. In order for a disjunction to be true, *at least one* of the components must be true. A disjunction is false only when *both* components are false. The truth table for the disjunction $p \vee q$ is given in Figure 1.20.

	p	q
1.	T	T
2.	T	F
3.	F	T
4.	F	F

Figure 1.18

Truth Values for Two Statements

	p	q	$p \wedge q$
1.	T	T	T
2.	T	F	F
3.	F	T	F
4.	F	F	F

Figure 1.19

Truth Table for a Conjunction $p \wedge q$

	p	q	$p \vee q$
1.	T	T	T
2.	T	F	T
3.	F	T	T
4.	F	F	F

Figure 1.20

Truth Table for a Disjunction $p \vee q$

∞ EXAMPLE 1 ∞ Construct a truth table for the compound statement $q \vee (p \wedge \sim q)$.

∞ SOLUTION ∞ Because there are two letters, we need $2 \cdot 2 = 4$ rows.

We need to insert a column for each connective in the compound statement $q \vee (p \wedge \sim q)$. As in algebra, we start inside any grouping symbols and work our way out. Therefore, we need a column for $\sim q$, a column for $p \wedge \sim q$, and a column for the entire compound statement $q \vee (p \wedge \sim q)$, as shown in Figure 1.21.

Figure 1.21

	p	q	$\sim q$	$p \wedge \sim q$	$q \vee (p \wedge \sim q)$
1.	T	T			
2.	T	F			
3.	F	T			
4.	F	F			

In the $\sim q$ column, fill in truth values that are opposite those for q. Next, the conjunction $p \wedge \sim q$ is true only when both components are true; enter a T in row 2 and Fs elsewhere. Finally, the disjunction $q \vee (p \wedge \sim q)$ is false only when both components q and $(p \wedge \sim q)$ are false; enter an F in row 4 and Ts elsewhere. The completed truth table is shown in Figure 1.22.

Figure 1.22
Truth Table for $q \vee (p \wedge \sim q)$

	p	q	$\sim q$	$p \wedge \sim q$	$q \vee (p \wedge \sim q)$
1.	T	T	F	F	T
2.	T	F	T	T	T
3.	F	T	F	F	T
4.	F	F	T	F	F

If the symbolic representation of a compound statement consists of two different letters, its truth table will have $2 \cdot 2 = 4$ rows. How many rows are required if a compound statement consists of three letters, say p, q, and r? Because each statement has two possible truth values (T and F), the truth table must contain $2 \cdot 2 \cdot 2 = 8$ rows. In general, each time a new statement is added, the number of rows doubles.

Number of Rows

If a compound statement consists of *n* individual statements, each represented by a different letter, the number of rows required in its truth table is 2^n.

∞ EXAMPLE 2 ∞

Under what conditions is the compound statement $p \wedge \sim(q \vee r)$ true?

∞ SOLUTION ∞

We can answer the question by constructing a truth table. Since there are three letters, we need $2^3 = 8$ rows. We start with three columns, one for each letter. In order to account for all possible combinations of *p*, *q*, and *r* as true or false, proceed as follows:

	p	*q*	*r*
1.	T		
2.	T		
3.	T		
4.	T		
5.	F		
6.	F		
7.	F		
8.	F		

(a)

	p	*q*	*r*
1.	T	T	
2.	T	T	
3.	T	F	
4.	T	F	
5.	F	T	
6.	F	T	
7.	F	F	
8.	F	F	

(b)

	p	*q*	*r*
1.	T	T	T
2.	T	T	F
3.	T	F	T
4.	T	F	F
5.	F	T	T
6.	F	T	F
7.	F	F	T
8.	F	F	F

(c)

Figure 1.23

Truth Values for Three Statements

1. Fill the first half (four rows) of column 1 with Ts and the rest with Fs, as shown in Figure 1.23(a).
2. In the next column, split each half into halves, with the first half receiving Ts and the second Fs. In other words, alternate two Ts and two Fs in column 2, as shown in Figure 1.23(b).
3. Again, split each half into halves; the first half receives Ts, and the second receives Fs. Because we are dealing with the third (last) column, the Ts and Fs will alternate, as shown in Figure 1.23(c).

(This process of filling the first half of the first column with Ts and the second half with Fs and then splitting each half into halves with blocks of Ts and Fs applies to all truth tables.)

We need to insert a column for each connective in the compound statement $p \wedge \sim(q \vee r)$, as shown in Figure 1.24.

Figure 1.24

	p	q	r	$q \vee r$	$\sim(q \vee r)$	$p \wedge \sim(q \vee r)$
1.	T	T	T			
2.	T	T	F			
3.	T	F	T			
4.	T	F	F			
5.	F	T	T			
6.	F	T	F			
7.	F	F	T			
8.	F	F	F			

Now fill in the appropriate symbol in the column under $q \vee r$. Enter *F* if *both* q and r are F; enter T otherwise (at least one is true). In the $\sim(q \vee r)$ column, fill in truth values that are opposite those for $q \vee r$, as shown in Figure 1.25.

Figure 1.25

	p	q	r	$q \lor r$	$\sim(q \lor r)$	$p \land \sim(q \lor r)$
1.	T	T	T	T	F	
2.	T	T	F	T	F	
3.	T	F	T	T	F	
4.	T	F	F	F	T	
5.	F	T	T	T	F	
6.	F	T	F	T	F	
7.	F	F	T	T	F	
8.	F	F	F	F	T	

The conjunction $p \land \sim(q \lor r)$ is true only when *both* p and $\sim(q \lor r)$ are true; enter a T in row 4 and Fs elsewhere. The completed truth table is shown in Figure 1.26.

Figure 1.26

Truth Table for $p \land \sim(q \lor r)$

	p	q	r	$q \lor r$	$\sim(q \lor r)$	$p \land \sim(q \lor r)$
1.	T	T	T	T	F	F
2.	T	T	F	T	F	F
3.	T	F	T	T	F	F
4.	T	F	F	F	T	T
5.	F	T	T	T	F	F
6.	F	T	F	T	F	F
7.	F	F	T	T	F	F
8.	F	F	F	F	T	F

As indicated in the truth table, the compound statement $p \wedge \sim(q \vee r)$ is true only when p is true and both q and r are false.

THE CONDITIONAL $p \rightarrow q$

A **conditional** is a compound statement of the form "if p then q" and is symbolized $p \rightarrow q$. Under what circumstances is a conditional true, and when is it false? Consider the following (compound) statement: "If you give me \$50, then I will give you a ticket to the ballet." This statement is a conditional and has the following representation:

p: You give me \$50,

q: I give you a ticket to the ballet,

$p \rightarrow q$: If you give me \$50, then I will give you a ticket to the ballet.

The conditional can be viewed as a promise: *If* you give me \$50, *then* I will give you a ticket to the ballet. Suppose you give me \$50; that is, suppose p is true. I have two options: either I give you a ticket to the ballet (q is true), or I do not (q is false). If I do give you the ticket, the conditional $p \rightarrow q$ is true (I have kept my promise); if I do not give you the ticket, the conditional $p \rightarrow q$ is false (I have not kept my promise). These situations are shown in rows 1 and 2 of the truth table in Figure 1.27. Rows 3 and 4 require further analysis.

	p	q	$p \rightarrow q$
1.	T	T	T
2.	T	F	F
3.	F	T	?
4.	F	F	?

Figure 1.27

Suppose you do not give me \$50; that is, suppose p is false. Regardless of whether or not I give you a ticket, you cannot say that I broke my promise; that is, you cannot say that the conditional $p \rightarrow q$ is false. Consequently, since a statement is either true or false, the conditional is labeled true (by default). In other words, when the premise p of a conditional is false, it does not matter whether the conclusion q is true or false. In both cases, the conditional $p \rightarrow q$ is automatically labeled true, because it is not false.

The completed truth table for a conditional is given in Figure 1.28. Notice that the only circumstance under which a conditional is false is when the premise p is true and the conclusion q is false, as shown in row 2.

	p	q	$p \rightarrow q$
1.	T	T	T
2.	T	F	F
3.	F	T	T
4.	F	F	T

Figure 1.28

Truth Table for a Conditional $p \rightarrow q$

∞ EXAMPLE 3 ∞

Under what conditions is the compound statement $q \rightarrow \sim p$ true?

∞ SOLUTION ∞

Our truth table has $2^2 = 4$ rows and contains a column for p, q, $\sim p$, and $q \rightarrow \sim p$, as shown in Figure 1.29.

In the $\sim p$ column, fill in truth values that are opposite those for p. Now, a conditional is false only when its premise (in this case q) is true and its conclusion (in this case $\sim p$) is false. Therefore, $q \rightarrow \sim p$ is false only in row 1; the conditional $q \rightarrow \sim p$ is true under all conditions except the condition that both p and q are true. The completed truth table is shown in Figure 1.30.

	p	q	$\sim p$	$q \rightarrow \sim p$
1.	T	T		
2.	T	F		
3.	F	T		
4.	F	F		

Figure 1.29

	p	q	$\sim p$	$q \rightarrow \sim p$
1.	T	T	F	F
2.	T	F	F	T
3.	F	T	T	T
4.	F	F	T	T

Figure 1.30

Truth Table for $q \rightarrow \sim p$

∞ EXAMPLE 4 ∞ Construct a truth table for the following compound statement: "I walk up the stairs if I want to exercise or if the elevator isn't working."

∞ SOLUTION ∞ Rewriting the statement so the word *if* is first, we have "If I want to exercise or (if) the elevator isn't working, then I walk up the stairs."

Now we must translate the statement into symbols and construct a truth table.

Define the following:

p: I want to exercise,

q: The elevator is working,

r: I walk up the stairs.

The statement now has the symbolic representation $(p \vee \sim q) \rightarrow r$. Because we have three letters, our table must have $2^3 = 8$ rows. Inserting a column for each letter and a column for each connective, we have the initial setup shown in Figure 1.31.

Figure 1.31

	p	q	r	$\sim q$	$p \vee \sim q$	$(p \vee \sim q) \rightarrow r$
1.	T	T	T			
2.	T	T	F			
3.	T	F	T			
4.	T	F	F			
5.	F	T	T			
6.	F	T	F			
7.	F	F	T			
8.	F	F	F			

In the column labeled $\sim q$, enter truth values that are the opposite of those of q. Next enter the truth values of the disjunction $p \vee \sim q$ in column 5. Recall that a disjunction is false only when both components are false and is true otherwise. Consequently, enter Fs in rows 5 and 6 (since both p and $\sim q$ are false) and Ts in the remaining rows, as shown in Figure 1.32.

Figure 1.32

	p	q	r	$\sim q$	$p \vee \sim q$	$(p \vee \sim q) \rightarrow r$
1.	T	T	T	F	T	
2.	T	T	F	F	T	
3.	T	F	T	T	T	
4.	T	F	F	T	T	
5.	F	T	T	F	F	
6.	F	T	F	F	F	
7.	F	F	T	T	T	
8.	F	F	F	T	T	

The last column involves a conditional; it is false only when its premise is true and its conclusion is false. Therefore, enter Fs in rows 2, 4, and 8, since $p \vee \sim q$ is true and r is false, and enter Ts in the remaining rows. The final truth table is shown in Figure 1.33.

Figure 1.33

Truth Table for $(p \vee \sim q) \rightarrow r$

	p	q	r	$\sim q$	$p \vee \sim q$	$(p \vee \sim q) \rightarrow r$
1.	T	T	T	F	T	T
2.	T	T	F	F	T	F
3.	T	F	T	T	T	T
4.	T	F	F	T	T	F
5.	F	T	T	F	F	T
6.	F	T	F	F	F	T
7.	F	F	T	T	T	T
8.	F	F	F	T	T	F

EQUIVALENT STATEMENTS

When you purchase a car, the car is either new or used. If a salesperson told you that "it is not the case that the car is not new," what condition would the car be in? This compound statement consists of one individual statement ("*p:* the car is new") and two negations:

"It is not the case that the car is not new."

$\sim$ $\sim p$

Does this mean that the car is new? To answer this question, we will construct a truth table for the compound statement $\sim(\sim p)$ and compare its truth values with those of the original statement, p. Because there is only one letter, we need $2^1 = 2$ rows, as shown in Figure 1.34.

Figure 1.34

	p
1.	T
2.	F

We must insert a column for $\sim p$ and a column for $\sim(\sim p)$. Now $\sim p$ has truth values that are opposite those of p, and $\sim(\sim p)$ has truth values that are opposite those of $\sim p$, as shown in Figure 1.35.

	p	$\sim p$	$\sim(\sim p)$
1.	T	F	T
2.	F	T	F

Figure 1.35

Truth Table for $\sim(\sim p)$

Notice that the values in the column labeled $\sim(\sim p)$ are identical to those in the column labeled p. Whenever this happens, the statements are said to be equivalent and may be used interchangeably. Therefore, the statement "it is not the case that the car is not new" is equivalent in meaning to the statement "the car is new."

Equivalent statements are statements that have identical truth table values in each corresponding entry. The expression $p \equiv q$ is read "p is equivalent to q" or "p and q are equivalent." As we can see in Figure 1.35, a statement and its double negation are logically equivalent. This relationship can be expressed as $p \equiv \sim(\sim p)$.

∞ EXAMPLE 5 ∞

Are the statements "If Thurston Howell is elected, taxes go up" and "Thurston Howell is elected and taxes do not go up" equivalent?

∞ SOLUTION ∞

We begin by defining the statements:

p: Thurston Howell is elected,

q: Taxes go up,

$p \rightarrow q$: If Thurston Howell is elected, taxes go up,

$p \wedge \sim q$: Thurston Howell is elected and taxes do not go up.

The truth table contains $2^2 = 4$ rows, and the initial setup is shown in Figure 1.36.

Figure 1.36

	p	q	$\sim q$	$p \wedge \sim q$	$p \to q$
1.	T	T			
2.	T	F			
3.	F	T			
4.	F	F			

Now enter the appropriate truth values under $\sim q$ (the opposite of q). Because the conjunction $p \wedge \sim q$ is true only when both p and $\sim q$ are true, enter a T in row 2 and Fs elsewhere. The conditional $p \to q$ is false only when p is true and q is false; therefore, enter an F in row 2 and Ts elsewhere. The completed truth table is shown in Figure 1.37.

Figure 1.37

	p	q	$\sim q$	$p \wedge \sim q$	$p \to q$
1.	T	T	F	F	T
2.	T	F	T	T	F
3.	F	T	F	F	T
4.	F	F	T	F	T

Because the entries in the columns labeled $p \wedge \sim q$ and $p \to q$ are not the same, the statements are not equivalent. "If Thurston Howell is elected, taxes go up" is *not* equivalent to "Thurston Howell is elected and taxes do not go up."

Notice that the truth values in the columns under $p \wedge \sim q$ and $p \to q$ in Figure 1.37 are exact opposites; when one is T, the other is F. Whenever this happens, one statement is the negation of the other. Consequently, $p \wedge \sim q$ is the negation of $p \to q$ (and vice versa). This can be expressed as $p \wedge \sim q \equiv \sim(p \to q)$. The negation of a conditional is logically equivalent to the conjunction of the premise and the negation of the conclusion.

+ Historical Note +

= GEORGE BOOLE =
(1815–1864)

George Boole is called "the father of symbolic logic." Computer science owes much to this self-educated mathematician. Born the son of a poor shopkeeper, Boole had very little formal education, and his prospects for rising above his family's lower-class status were dim. Like Leibniz, he taught himself Latin; at the age of twelve, he translated an ode of Horace into English, winning the attention of the local schoolmasters. (In his day, the knowledge of Latin was a prerequisite to scholarly endeavors and to becoming a socially accepted gentleman.) After that, his academic desires were encouraged, and at the age of fifteen he began his long teaching career. While teaching arithmetic, he studied advanced mathematics and physics.

In 1849, after nineteen years of teaching at elementary schools, Boole received his big break—he was appointed professor of mathematics at Queen's College in the city of Cork, Ireland. At last he was able to research advanced mathematics, and he became recognized as a first-class mathematician. This was a remarkable feat, considering Boole's lack of formal training and degrees.

Boole's most influential work, *An Investigation of the Laws of Thought, on Which Are Founded the Mathematical Theories of Logic and Probabilities,* was published in 1854. In it he wrote "There exist certain general principles founded in the very nature of language and logic that exhibit laws as identical in form as with the laws of the general symbols of algebra." With this insight, Boole had taken a big step into the world of logical reasoning and abstract mathematical analysis.

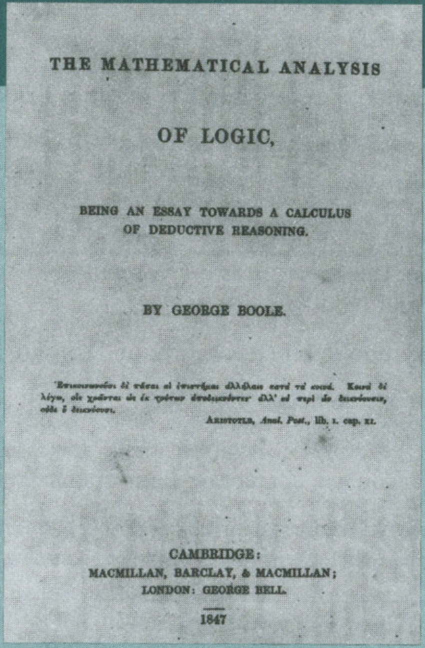
Perhaps because of his lack of formal training, Boole challenged the status quo, including the Aristotelian assumption that *all* logical arguments could be

Statements that look or sound different may in fact have the same meaning. For example, "It is not the case that the car is not new" really means the same as "The car is new," and "It is not the case that if Thurston Howell is elected taxes go up" actually means the same as "Thurston Howell is elected and taxes do not go up." When we are working with equivalent statements, we can substitute either statement for the other without changing the truth value.

DE MORGAN'S LAWS

Earlier in this section, we saw that the negation of a negation is equivalent to the original statement; that is, $\sim(\sim p) \equiv p$. Another negation "formula" we discovered was $\sim(p \rightarrow q) \equiv p \wedge \sim q$, that is, the negation of a conditional. Can we find similar "formulas" for the negations of the other basic connecreduced to syllogistic arguments. In doing so, he employed symbols to represent concepts, as did Leibniz, but he also developed systems of algebraic manipulation to accompany these symbols. Thus, Boole's creation is a marriage of logic and mathematics. However, as is the case with almost all new theories, Boole's symbolic logic was not met with total adulation. In particular, one staunch opponent of his work was Georg Cantor, whose work on the origins of set theory and the magnitude of infinity will be investigated in Chapter 2.

In the many years since Boole's original work was unveiled, various scholars have modified, improved, generalized, and extended its central concepts. Today, Boolean algebras are the essence of computer software and circuit design. After all, a computer merely manipulates predefined symbols and conforms to a set of preassigned algebraic commands.


THE MATHEMATICAL ANALYSIS

OF LOGIC,

BEING AN ESSAY TOWARDS A CALCULUS
OF DEDUCTIVE REASONING.

BY GEORGE BOOLE.

Ἐπικοινωνοῦσι δὲ πᾶσαι αἱ ἐπιστῆμαι ἀλλήλαις κατὰ τὰ κοινά. Κοινὰ δὲ λέγω, οἷς χρῶνται ὡς ἐκ τούτων ἀποδεικνύντες· ἀλλ' οὐ περὶ ὧν δεικνύουσιν, οὐδὲ ὃ δεικνύουσι.

ARISTOTLE, *Anal. Post.*, lib. I. cap. XI.

CAMBRIDGE:
MACMILLAN, BARCLAY, & MACMILLAN;
LONDON: GEORGE BELL.

1847

Through an algebraic manipulation of logical symbols, Boole revolutionized the age-old study of logic. His essay *The Mathematical Analysis of Logic* laid the foundation for his later book *An Investigation of the Laws of Thought.*

tives, namely, the conjunction and the disjunction? The answer is yes, and the results are credited to the English mathematician and logician Augustus De Morgan.

De Morgan's Laws

The negation of the conjunction $p \wedge q$ is given by $\sim(p \wedge q) \equiv \sim p \vee \sim q$.
"Not p and q" is equivalent to "not p or not q."

The negation of the disjunction $p \vee q$ is given by $\sim(p \vee q) \equiv \sim p \wedge \sim q$.
"Not p or q" is equivalent to "not p and not q."

De Morgan's Laws are easily verified through the use of truth tables and will be addressed in the exercises (see Exercises 37 and 38).

∞ **EXAMPLE 6** ∞

Using De Morgan's Laws, find the negation of each of the following.

a. It is Friday and I just got paid.
b. You are correct or I am crazy.

∞ **SOLUTION** ∞

a. The symbolic representation of "It is Friday and I just got paid" is

p: It is Friday,

q: I just got paid,

$p \wedge q$: It is Friday and I just got paid.

Therefore, the negation is $\sim(p \wedge q) \equiv \sim p \vee \sim q$, that is, "It is not Friday or I did not get paid."

b. The symbolic representation of "You are correct or I am crazy" is

p: You are correct,

q: I am crazy,

$p \vee q$: You are correct or I am crazy.

Therefore, the negation is $\sim(p \vee q) \equiv \sim p \wedge \sim q$, that is, "You are not correct and I am not crazy."

As we have seen, the truth value of a compound statement depends on the truth values of the individual statements that comprise it. The truth tables of the basic connectives are summarized in Figure 1.38.

p	$\sim p$
T	F
F	T

Negation

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Conjunction

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Disjunction

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Conditional

Figure 1.38

Truth Tables for the Basic Connectives

Equivalent statements are statements that have the same meaning. Equivalent statements for the negations of the basic connectives are given in Figure 1.39.

Figure 1.39

Negations of the Basic Connectives

1.	$\sim(\sim p) \equiv p$	the negation of a negation
2.	$\sim(p \wedge q) \equiv \sim p \vee \sim q$	the negation of a conjunction
3.	$\sim(p \vee q) \equiv \sim p \wedge \sim q$	the negation of a disjunction
4.	$\sim(p \rightarrow q) \equiv p \wedge \sim q$	the negation of a conditional

+ EXERCISES 1.3 +

In Exercises 1–20, construct a truth table for the compound statement.

1. $p \vee \sim q$
2. $p \wedge \sim q$
3. $p \vee \sim p$
4. $p \wedge \sim p$
5. $p \rightarrow \sim q$
6. $\sim p \rightarrow q$
7. $\sim q \rightarrow \sim p$
8. $\sim p \rightarrow \sim q$
9. $(p \vee q) \rightarrow \sim p$
10. $(p \wedge q) \rightarrow \sim q$
11. $(p \vee q) \rightarrow (p \wedge q)$
12. $(p \wedge q) \rightarrow (p \vee q)$
13. $p \wedge \sim(q \vee r)$
14. $p \vee \sim(q \vee r)$
15. $p \vee (\sim q \wedge r)$
16. $\sim p \vee \sim(q \wedge r)$
17. $(\sim r \vee p) \rightarrow (q \wedge p)$
18. $(q \wedge p) \rightarrow (\sim r \vee p)$
19. $(p \vee r) \rightarrow (q \wedge \sim r)$
20. $(p \wedge r) \rightarrow (q \vee \sim r)$

In Exercises 21–28, translate the compound statement into symbolic form and then construct the truth table for the statement.

21. If it doesn't rain, we will ration the water supply.
22. We are in trouble if he is elected.
23. If he doesn't say hello, he's late or mad.
24. They are in big trouble if they go out and are seen together.
25. If Gene goes to the golf course, we won't go to the museum and Karen will be upset.
26. If you use leaded gasoline, the catalytic converter will be damaged and it will need to be replaced.
27. If Proposition A passes and Proposition B does not, the legislature will not have an easy time allocating funds and new taxes will be imposed.
28. If Proposition A doesn't pass and the legislature raises taxes, the governor will veto the action and promote his own plan.

In Exercises 29–36, construct a truth table to determine whether the statements in each pair are equivalent.

29. The food is inexpensive or good.
If the food is expensive, then it is good.
30. The food is inexpensive or good.
If the food is not good, then it is inexpensive.
31. The speaker is sincere or I am crazy.
If the speaker is not sincere, then I am crazy.
32. The speaker is not telling the truth or I am crazy.
If I am not crazy, then the speaker is not telling the truth.
33. If candidate X is elected, I will move to another state.
If I move to another state, candidate X is elected.
34. If candidate X is elected, I will move to another state.
If candidate X is not elected, I will stay in this state.
35. She donated blood or did not make a cash contribution.
She made a cash contribution and did not donate blood.
36. He did not donate blood and he did not make a cash contribution.
It is not the case that he donated blood or made a cash contribution.
37. Using truth tables, verify De Morgan's Law $\sim(p \wedge q) \equiv \sim p \vee \sim q$.

Logic Formulas

We now list logic formulas including those we have already proved. We will not prove any of the additional formulas, but they may be used as optional exercises.

1. $P \Rightarrow P$
2. $P \vee P \Leftrightarrow P$
3. $P \wedge P \Leftrightarrow P$
4. $P \wedge Q \Rightarrow P$
5. $P \Rightarrow P \vee Q$
6. $P \wedge Q \Leftrightarrow Q \wedge P$
7. $P \vee Q \Leftrightarrow Q \vee P$
8. $[(P \vee Q) \vee R] \Leftrightarrow [P \vee (Q \vee R)]$
9. $[(P \wedge Q) \wedge R] \Leftrightarrow [P \wedge (Q \wedge R)]$
10. $[P \vee (Q \wedge R)] \Leftrightarrow [(P \vee Q) \wedge (P \vee R)]$
11. $[P \wedge (Q \vee R(] \Leftrightarrow [(P \wedge Q) \vee (P \wedge R)]$
12. $[(P \to Q) \wedge (Q \to R)] \Rightarrow (P \to R)$
13. $(P \to Q) \Leftrightarrow (\sim Q \to \sim P)$ Contrapositive proof
14. $(P \to Q) \Leftrightarrow (\sim P \vee Q)$
15. $[(P \to Q) \wedge (Q \to P)] \Leftrightarrow (P \leftrightarrow Q)$
16. $[(P \leftrightarrow Q) \wedge (Q \leftrightarrow R)] \Rightarrow (P \leftrightarrow R)$
17. $[P \wedge (P \to Q)] \Rightarrow Q$
18. $[P \wedge (\sim P \vee Q)] \Rightarrow Q$
19. $[(P \to Q) \wedge (P \to R)] \Leftrightarrow [P \to (Q \wedge R)]$
20. $[(P \to Q) \vee (P \to R)] \Leftrightarrow [P \to (Q \vee R)]$
21. $[(P \to Q) \wedge (\sim P \to Q)] \Rightarrow Q$
22. $[(P \to Q) \wedge (R \to S)] \Rightarrow [(P \vee R) \to (Q \vee S)]$
23. $[(P \to Q) \wedge (R \to S)] \Rightarrow [(P \wedge R) \to (Q \wedge S)]$
24. $[(P \to Q) \vee (R \to S)] \Rightarrow [(P \wedge R) \to (Q \vee S)]$
25. $[(P \to R) \vee (Q \to R)] \Rightarrow [(P \wedge Q) \to R]$
26. $[(P \to R) \wedge (Q \to R)] \Rightarrow [(P \vee Q) \to R]$
27. $R \Rightarrow [(P \to Q) \leftrightarrow ((P \wedge \sim Q) \to (Q \vee (\sim P \vee \sim R)))]$
28. $\sim(\sim P) \Leftrightarrow P$
29. $\sim(P \vee Q) \Leftrightarrow (\sim P \wedge \sim Q)$
30. $\sim(P \wedge Q) \Leftrightarrow (\sim P \vee \sim Q)$
31. $\sim(P \to Q) \Leftrightarrow (P \wedge \sim Q)$ Indirect Proof contradiction