ACOUSTICAL EVOLUTION OF WIND INSTRUMENTS

A course (Physics 323) taught by A. H. Benade in the fall of 1977
Case Western University, Cleveland, Ohio

INTRODUCTION

It is the aim of this course to study the acoustical nature of flutes, oboes, bassoons, trumpets, horns, and clarinets as they have evolved from the baroque era to the present (the particular emphasis on the different instruments will depend on the special interests of students enrolled in the course).

While acoustical concepts will be used as a basis for understanding the instruments under study, the emphasis will be on matters of direct and practical interest to the performing musician, and thence also to the music historian. Topics for consideration will be of the following sort:

1. Evaluation and adjustment (of reed, etc.), proper embouchure selection, and other aspects of playing technique to best exploit the properties of an instrument (as governed by the individual instrument and traditional practices in the era of its normal usage).

2. Under the guidance of the foregoing, methods for the determination of the original tuning level (A-440, A-421, etc.) as indicated by best scale and best overall tonal response.

3. Recognition and causes of the strong and weak notes, note-by-note intonation behavior, tone-color tendencies, and rapidity and stability of speech on various notes of the scale, as these things influence the player and so also influenced the composer at the time of the instrument's major usage.

4. Implications of the foregoing for authentic and musically usable performances of baroque- and classical-era music today, on early instruments and on modern ones.

To the maximum extent possible, students will meet and play upon good instruments of the sort under discussion. Informal ensemble and solo performances on these instruments will be expected, along with a term paper on a topic chosen to suit the student's particular interests and needs.

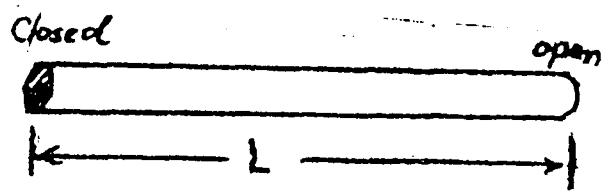
PREREQUISITES: Acoustics of Music 319, or consent of the instructor. Acquaintance with Chapters 15, 16, 20, 21, and 22 of AHB's <u>Fundamentals of Musical Acoustics</u>.

[Note: These pages have been typewritten from handwritten ditto copies of class handout sheets prepared for the students of this course. The dittos were faded, sometimes in the extreme, and occasionally material written close to the edge of the paper did not print on the ditto copy. Virginia Benade, 1989]

A. SOME PROPERTIES OF AIR COLUMNS

There are several general properties of air columns that underlie the use and adjustment of wind instruments which can usefully be sketched right at the beginning. The meaning and implications of these properties will become clearer as we work our way through the course.

We take as our point of departure the cylindrical pipe closed at one end, length L, radius a:



Natural frequencies of this pipe lie at

$$f_n = (2n - 1)\frac{c}{4L} \qquad (1)$$

where n = 1, 2, 3, . . . , which is the mode number; c = velocity of sound = 345 m/sec.

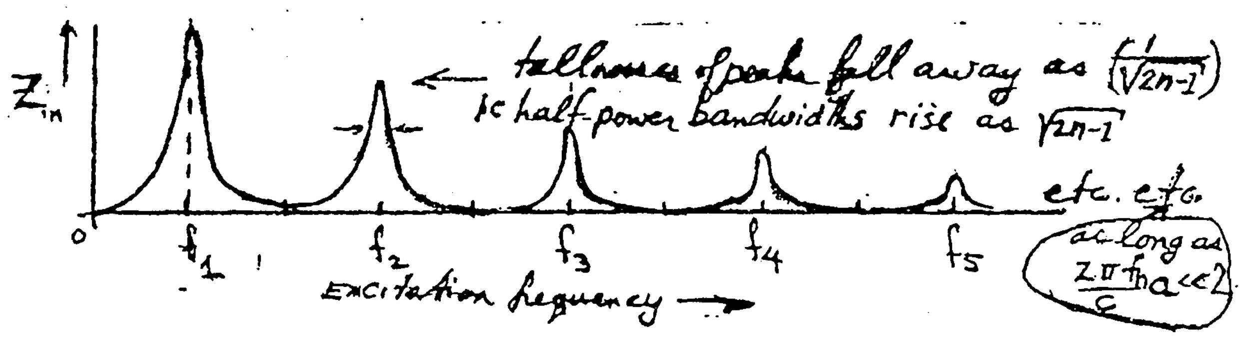
A pipe 1 meter long has frequencies of 86.25, 3 x 86.25 = 258.75, 5 x 86.25 = 431.25, . . . Hz.

Never mind about "end corrections," etc., because there are many things of this sort and we'd never get done . . . the most familiar one is, by the way, <u>not</u> the most important one!

Note: The eigenvalue equation from which this result is obtained is

$$\tan\frac{2\pi f_n}{c} L = 0 \qquad (2)$$

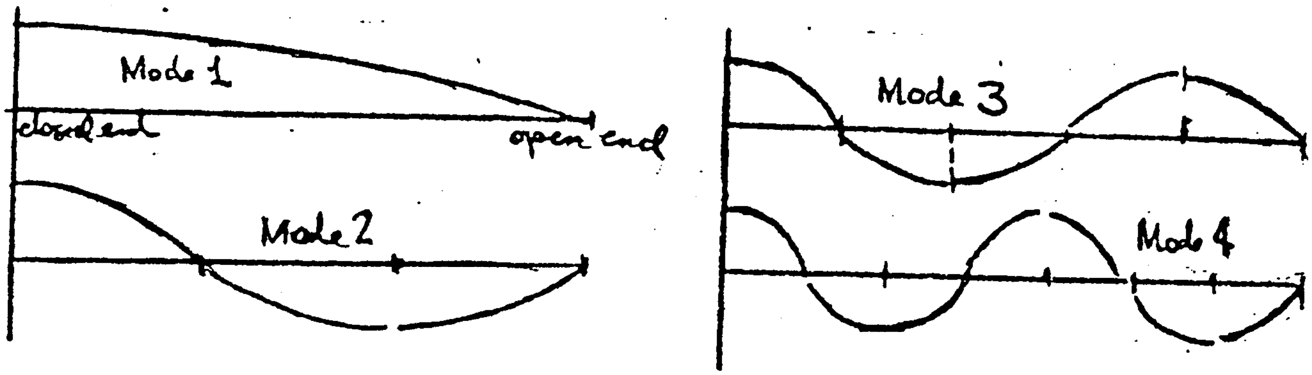
The resonance curve for input impedance measured for what we have called the closed end looks like this (see FMA Fig. 20.4, p. 398; Fig. 21.3, p. 435; also Fig. 20.3, p. 396):



For our reference pipe, the natural frequencies lie in a 1, 3, 5 sequence so they form the odd-numbered members of a harmonic series.

What is the musical interval name for $f_2/f_1 = 3/1$ = ?? (nontrivial for our uses). [More combinations were given, but part slipped off bottom of page: "for $f_3/f_2 = 5/3$ = ??; for $f_4/_3 = 7/5$ = ??"]

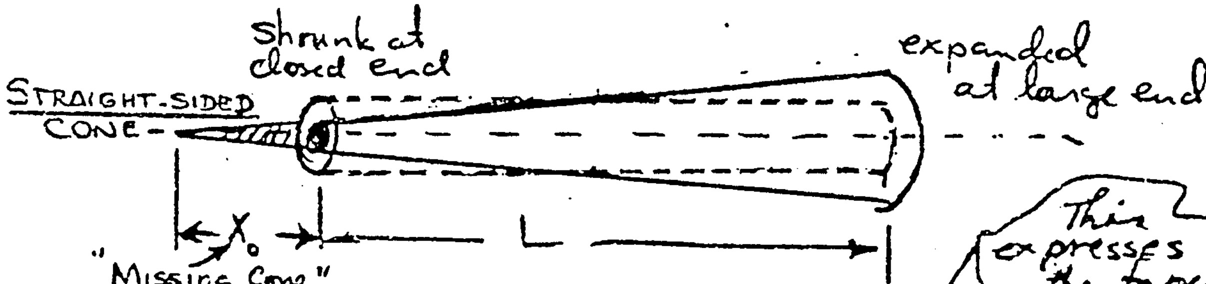
The pressure standing wave patterns belonging to these various natural frequencies are as follows:



$$P_n = A_n \cos\left[\left(\frac{(2n-1)\pi}{2}\right)\left(\frac{X}{L}\right)\right]$$

EACH standing wave has (2n - 1) half humps. In the case at hand, all the half humps for a given mode are alike--THIS IS NOT GENERALLY THE CASE!

FIRST VARIATION (The Expanding Cone)



This expresses the taper

~~Approx but good~~ See Morse Eq 24.23 page 286

$$f_n \cong \left(\frac{c}{4L}\right)\sqrt{(2n-1)^2 + \left(\frac{8}{\pi^2}\right)\left(\frac{L}{X_0}\right)}$$

$$= \left(\frac{c}{4L}\right)(2n-1)\sqrt{1 + \left(\frac{8}{\pi^2}\right)\left(\frac{L}{X_0}\right)\left(\frac{1}{2n-1}\right)^2}$$

$\left(\frac{c}{4L}\right)(2n-1)$: as for cyl!

$\left(\frac{8}{\pi^2}\right)\left(\frac{L}{X_0}\right)\left(\frac{1}{2n-1}\right)^2$: becomes negligible as n gets large

This formula works reasonably well as long as $X_0 \gtrsim (1/3)L$.

(a) Notice that all modes have their frequencies raised relative to their cylindrical ancestors. This is mildly important to us.

(b) Since the amount (ratio) of raising is less for the high than for the low modes, the musical intervals between successive natural frequencies are <u>compressed</u> relative to the reference series of odd harmonics. THIS COMPRESSION WILL BE EXCEEDINGLY IMPORTANT TO US!

For the scientific types in the class, I note in passing that f is found from the EXACT eigen value equation,

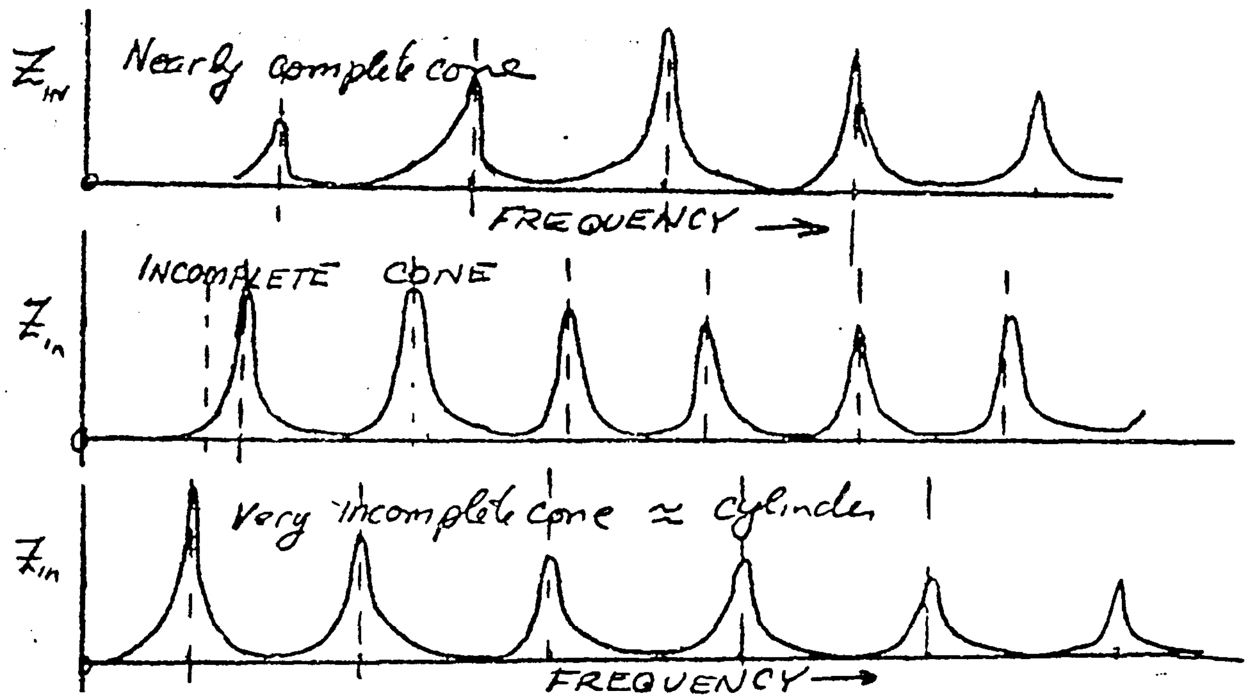
$$\tan \frac{2\pi f_n L}{f} + \frac{2\pi f_n X_o}{c} = 0$$

(For standing waves in a <u>complete</u> cone, see FMA Fig. 21.2, p. 433; see also Fig. 21.1.)

When $X_o \gtrsim L$, use the formula given already for f_n. When $X_o \lesssim L/2$, use the following formula:

$$f_n \cong \frac{nc}{2(L + X_o)}$$

FOR A NEARLY COMPLETE CONE, f_1 is nearly 2 x f_1 for the reference pipe and the f_n's form <u>very nearly</u> a complete harmonic series. This approximation is useful to know for us. Also, the approximation is not quite good enough for musical use!



[A 4]

VERY IMPORTANT FOR US: Peak 1 is not the tallest when the cone is nearly complete. Peak 1 is the tallest when the cone is very incomplete. This explains much about the tone-color and dynamic-range behavior of cone woodwinds.

MOST IMPORTANT: ENLARGING TOWARD THE OPEN END, OR CONTRACTING TOWARD THE CLOSED END

(a) shrinks f_{n+1}/f_n ratios

(b) raises f_1 most, higher f_n's less

CONTRACTING TOWARD OPEN END (etc., etc.) has opposite effect. These ideas underlie much of the basic adjustment of air column shapes.

This enlarging and contracting underlies much of the basic adjustment of air column shapes.

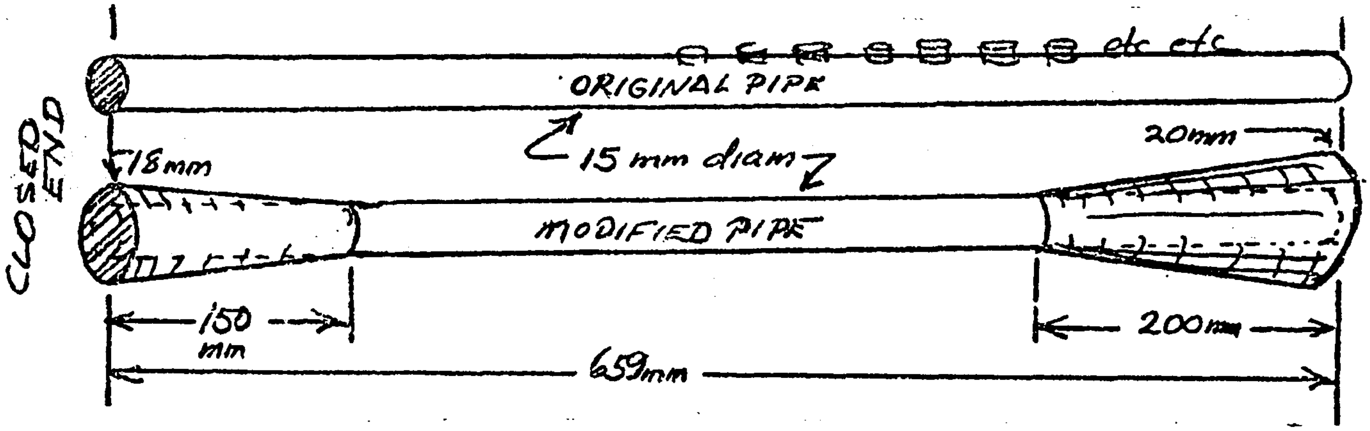
B. ASSIGNMENT 1

1. Consider a cylindrical pipe provided with a set of "idiot simple" toneholes which progressively saw off the tube at the following equivalent lengths:

659.3, 622.3, 587.4, 554.4, 323.3, 493.9, 466.2, 410.5,
415.4, 392.1, 370.0, 349.3, 329.7 (millimeters!)

Such a pipe (along with its toneholes) will have its first-mode natural frequencies in a chromatic scale from C3 to C4.

(a) State the note names belonging to the second-mode frequencies.



Science majors should use the perturbation integrals for these

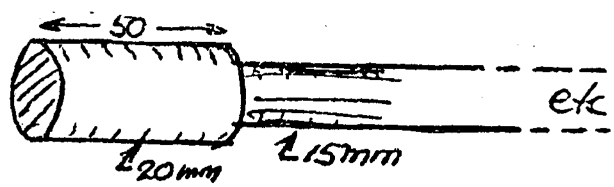
Suppose the original pipe is modified so that it expands a little at both ends, as shown.

(b) Tell what happens to the first-mode frequencies belonging to the tonehole systems for C3, G3, C4.

(c) Tell what happens to the musical intervals, and then the note names for the corresponding second-mode frequencies.

BE AS QUANTITATIVE AS POSSIBLE THROUGHOUT . . . BUT!!

2. Suppose we start out as before, with a pipe plus a series of idealized toneholes giving a first-mode sequence of frequencies whose note names run from C3 to C4. This time, however, the closed ("top," "north") end is enlarged over a 50-mm length, as sketched. This will change the first- and second-mode frequencies in a very similar fashion.



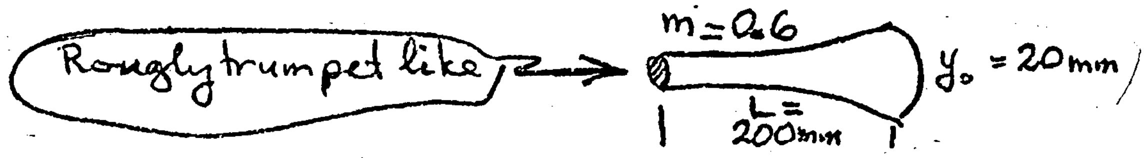
(a) Calculate the frequencies, and thence determine the note names for the modified system's first-mode oscillations.

(b) What would have been the frequencies (A_{new}/A_{old} = 4.5) if instead of a 5-mm diameter increase at the upper end, a stick had been poked up all but the top 50 mm of the original tube, the cross section of the stick being equal to the <u>additional</u> cross section $\pi(20^2 - 15^2)$ mm² provided by the enlargement sketched. IGNORE TONEHOLE COMPLICATIONS!

L_{eq} = 88.9 mm; 38.9 mm longer

$(\Delta f)/f = (\Delta L)/L \longrightarrow 38.9/659 \approx 0.06$

3. Think now about a Bessel-horn shape that is the prototype of the various brass-instrument air columns.



In FMA, sec. 20.5, p. 409, we find a formula for the natural frequencies for such a horn.

(a) Calculate the lengths L_p of cylindrical pipes that would have first-mode frequencies equal to the lowest three for this horn. This calculation is of the sort that was done in the planning of Fig. 20.8.

(b) Suppose some 20 to 30 percent additional length of cylindrical pipe is spliced into the middle of our horn. What would be the effect of this on the intermode frequency ratios?

(c) Why could one <u>not</u> simply splice the mouthpiece L_{eq} (FMA, Fig. 20.12, p. 415) for a trumpet onto the horn's own mechanical length in the formula referred to above in calculating the natural frequencies of a trumpet?

[The following appear in AHB's lecture copy]

by 2(b): much <u>greater</u> effect L_{eq} = 50 x 4.5 = 225 mm!

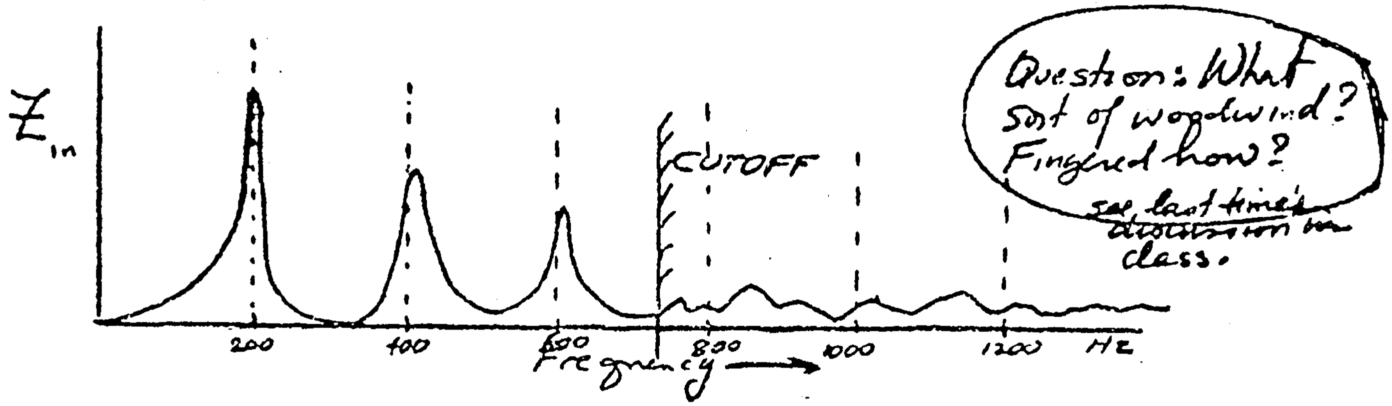
and by 3(a): f_1 = 392 x 1.624 = 636.7 Hz

f_2 = 392 x 3.624 = 1420.6 Hz

f_3 = 2204.6 Hz

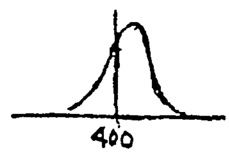
C. REGIMES OF OSCILLATION--PLAYING LEVEL (AT FIRST, FOR THE LOW REGISTER ONLY)

CASE I. Consider a single-reed woodwind having a resonance curve (of the sort sketched) for one of its notes. Assume peak 1 is at 200 Hz and peak 2 is at about 410 Hz; peak 3 is at 610 Hz, and the tonehole cutoff frequency is about 750 Hz. (Compare with discussion of Fig. 21.3 in FMA, p. 435).



(A) Pianissimo playing: The tallest peak runs the show, playing frequency is very close to 200 Hz and the internal spectrum consists of almost nothing but a 200-Hz sinusoid.

(B) Piano playing: The reed nonlinearity generates a little 2nd harmonic (about 400 Hz) which "talks" to peak 2. Clearly, more energy would be produced at this part of the spectrum if 2 times the playing frequency were nearly 410 Hz instead of at 400 Hz. Negotiations result in a slight rise in the playing frequency, to about 203 or 204 Hz. This is still pretty well on top of the peak-1 resonance curve, so the energy production here is almost as good ad before, but the second-harmonic component is now sitting almost smack on top of the peak of resonance 2, thus increasing the energy production.



Consequences: Playing frequency rises.
Second harmonic is strengthened considerably over the mere heterodyne production because of contribution via peak 2 to the energy budget.

Note that $f_p + 2f_p$ gives
- f_p ← transfer of energy between the two lower components
- $3f_p$ ← production of some 3rd harmonic over and above the tiny bit of 3f produced directly from f_1.

ALSO: The playing pitch is a little steadier than at the pp level, since there are now two air column resonances telling the reed what to do.

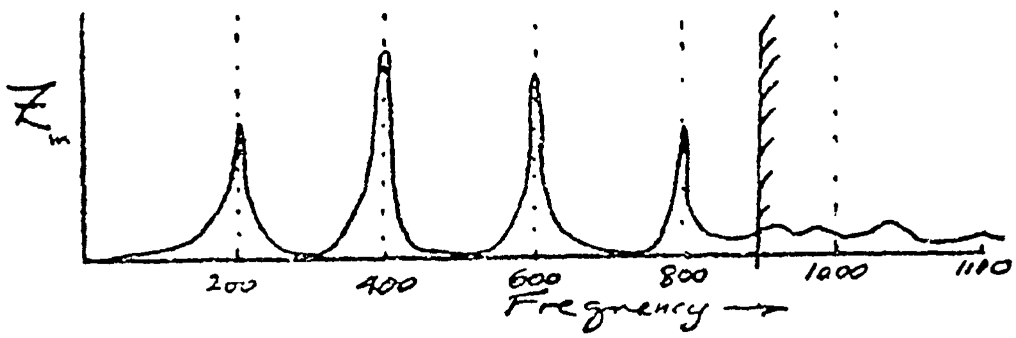
(C) Mezzopiano to mezzoforte playing: Enough 3rd harmonic is present that it talks appreciably to the air column resonance peak 3. Notice that for our particular instrument, the (3 x 203 Hz) component that goes with happy housekeeping for the lower two components and their corresponding peaks is perfectly in accord with the 610-Hz location of peak 3. The tone gets even better stabilized as a result, since 3 peaks are telling the reed what to do, in pretty good agreement. (Note: The saxophone tends to have its tone leap into existence when one tries a slow crescendo from ppp. A very similar discussion for brass instruments is to be found in FMA, p. 402, sec. 20.4. The related material on woodwinds is in sec. 21.3, p. 439.)

The internal spectrum then consists of a great deal of the 203-Hz fundamental component of the played note, a considerable amount of the 406-Hz second harmonic, and some significant amounts of the 609-Hz third harmonic.

All these components generate energy in cooperation with their respective resonances and pass it around among themselves, AND ALSO, VIA HETERODYNE ACTION, COMPONENTS APPEAR at 812 Hz (via doubling of component 2, adding of components 1 and 3, quadrupling of component 1); at 1015 Hz (via adding of components 2 and 3, doubling of component 1 added to component 3, doubling of component 3 minus component 1, etc., etc.); at 1218 Hz; etc. etc.

These are components that lie above cutoff and so do not have peaks to talk to, and as a result do not produce energy. They constitute a drag on the system, and so act to keep the playing level from rising further when the player tries to blow harder. NOTE! There is an important exception to the energy-loss behavior, which is associated with the reed resonance. We will postpone our discussion of it till we consider the second playing register.

CASE II. Let us turn our attention next to what happens when the second resonance peak is the tallest, again using a single reed.



This case makes for extremely "blurty" behavior if one tries to start a note ppp and have it grow.

(A) If one plays a good, solid forte, things are pretty much as before--here the fundamental and harmonics through the fourth peak of the 200-Hz playing frequency are nourished by air column resonance peaks. Again, heterodyne-produced components above cutoff are weakly present and act as an energy drain on the oscillatory system.

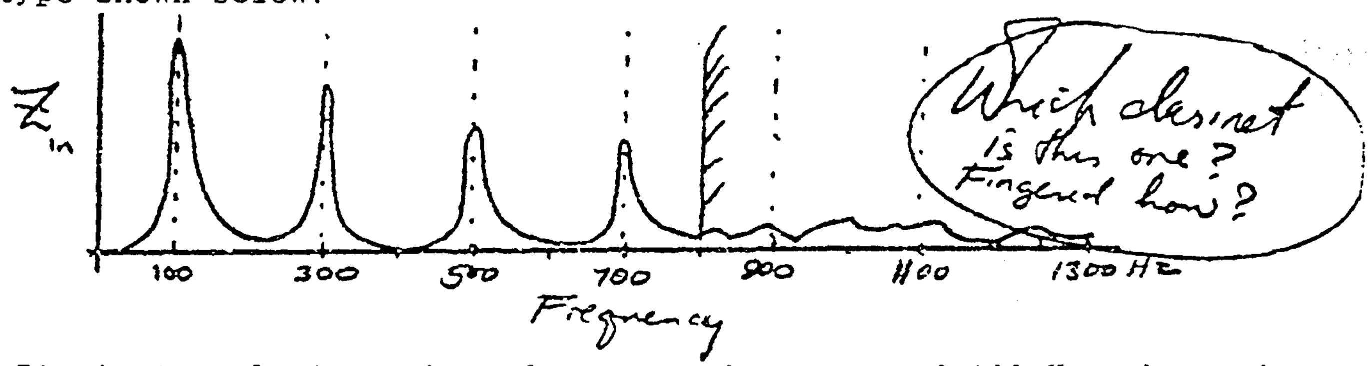
(B) We now play a diminuendo--and finally the cooperative energy input at

200, 400, 600, 800 Hz gets weak enough (and the nonlinear exchange of energy between components also) that the oscillation slips into one of a new type. The major input of energy is at 400 Hz (so the pitch rises an octave at the ppp level) and an attempt to play a crescendo causes a buildup of 2nd harmonic at 800 Hz, which is fed by peak 4. An attempt to push this 2-resonance regime of oscillation to higher dynamic levels "stalls" sooner than the one discussed earlier (based on 4 resonances) since there are less sources of energy and since it is already the third and higher harmonics that lie above cutoff and so constitute an increased energy drain.

This new regime of oscillation is a <u>second-register</u> regime for this instrument--we will look later into how it is routinely obtained at all dynamic levels in practical music.

Note! The "important exception" remarked on at the end of Case I has much greater significance for second-register regimes than for low-register ones.

<u>CASE III.</u> The third general case of interest has to do with the behavior of a reed coupled to a <u>cylindrical</u> pipe whose resonance curve might be of the type shown below.



(A) Pianissimo playing takes place at a frequency of 100 Hz, where the tallest peak is found. The internal spectrum is a very pure sinusoid.

(B) At a slightly increased dynamic level the reed produces some second harmonic at 200 Hz, which finds itself at a DIP in the resonance curve. Energy is therefore <u>absorbed</u> from the oscillation, so that the generated signal strength does not grow very much for a given increase in blowing pressure. There is only a little 2nd harmonic in the spectrum.

(C) Blowing harder yet makes the reed generate some 3rd harmonic by way of its nonlinear heterodyne action. This 300-Hz component finds an air column resonance peak to negotiate with, and so there is an input of energy at this frequency. The loudness of the signal therefore grows as the additional oscillatory help enters the picture.

(D) Further along in the crescendo we see again an increase of energy drain at 400 Hz followed by additional energy input at 500 Hz, drain at 600, then more help at 700 Hz--finally we reach a level at which any further increase is at the expense of progressively heavier damping via above-cutoff components.

NOTE: "The clarinet is expressive." Crescendo is extremely easy to control on the clarinet because of its alternating checks and balances of the above type.

WIND INSTRUMENT SOUND PRODUCTION

Every wind instrument consists of an air column coupled via a flow-controlling valve to the player's lungs, which serve as the primary source of compressed air. The system is kept in oscillation by a feedback loop in which acoustical disturbances in the mouthpiece operate the flow controller, and the resulting flow (which is a function of static and dynamic properties of the controller) serves as an excitory stimulus which produces oscillations of the air column.

In reed woodwinds the acoustic pressure in the mouthpiece or reed cavity operates the reed in its function as flow controller. The brass player's lips also function as a pressure-operated flow controller. In flutes the controller is velocity operated, the player's air jet being steered by the acoustic flow itself. (The excitation mechanism of the violin family is also of the velocity-operated type.)

We can represent the pressure-controlled flow u(t) past the reed into a woodwind mouthpiece as a power series in the mouthpiece pressure p, the coefficients being determined by the measured flow-control characteristics of the reed for a given blowing pressure P exerted in the player's mouth. The last term in the series represents the effect of a Bernoulli force exerted on the reed:

$$u = Ap + Bp^2 + Cp^3 \ldots + \beta u^2 \qquad (1)$$

In the case of periodic motion both the mouthpiece pressure p and the resulting flow may be represented by a Fourier series whose components are harmonics of the playing frequency ω_o. The nth flow component is linked to its corresponding pressure component by two things: (a) by the flow control characteristic of the reed and (b) by the input impedance Z_n of the air column at the frequency $n\omega_o$ (as modified by the elastic termination provided by the reed itself). The problem of solving the resulting system of simultaneous nonlinear equations can be formidable, but the general results are easily summarized if the Bernoulli term in Eq. (1) is set equal to zero:

$$P_1 = \frac{|Z_1|[Bp_1p_2 + 3/4\ Cp_1^3 + \ldots]}{1 - |Z_1|[A + 2Bp_o + \ldots]} \qquad (2)$$

$$P_{n>1} = \frac{|Z_n|[Bp_1^n + \ldots]}{1 - |Z_n|[A + 2Bp_o + \ldots]} \qquad (3)$$

Equation (3) shows at once that the nth harmonic component of the sound pressure <u>within the mouthpiece</u> varies as the nth power of the fundamental pressure amplitude and that it is proportional to $|Z\ |$. Thus, at pianissimo levels, the wave form is a pure sinusoid, and it blossoms during a crescendo into something whose spectral envelope is well caricatured by the envelope of the input impedance curve.

This orderly evolution is <u>not</u> observed when the reed tip is strongly influenced by Bernoulli effects, as is the case for double reeds and certain single reed mouthpieces. Here the behavior is quite different, with a tendency toward greatly increased production of even harmonics. In every case, fortissimo playing causes the reed to close completely during a part of each cycle. When this happens, the spectral envelope acquires a formantlike shape whose zeros are determined by the duty[?] cycle of the input air flow.

We learn from Eq. (2) and confirm from Eq. (3) that stable oscillation near threshold requires that the product $Z_n A$ (of input impedance and reed transconductance) must be unity. If $Z_n > 1/A$, the energy output at the frequency $n\omega_0$ is greater than the losses. Due to the nonlinearity of the flow-control process, energy is exchanged between components by heterodyne motion, and the entire system is locked together into a multicomponent <u>regime of oscillation</u> whose repetition frequency is one that maximizes the net oscillatory energy production. It turns out that an instrument whose resonances are properly aligned in harmonic relationship for each note in its scale in one which the musician finds to be excellent in many respects: such an instrument blows easily, is stable in pitch, is easily controllable for dynamics, and in general starts quickly and cleanly. There is also less noise and "incidental FM' when resonances are accurately aligned. The flute and violin-family instruments are sustained in a similar way, excitation being favored at the driving point admittance rather than at the impedance maxima, and once again the alignment of resonances is important in musicians' estimates of the quality.

When account is taken of the natural frequency ω_r of the reed valve, we find that the flow-control coefficients A, B, C, . . . become frequency-dependent. Thus, if A is the low-frequency limit of A,

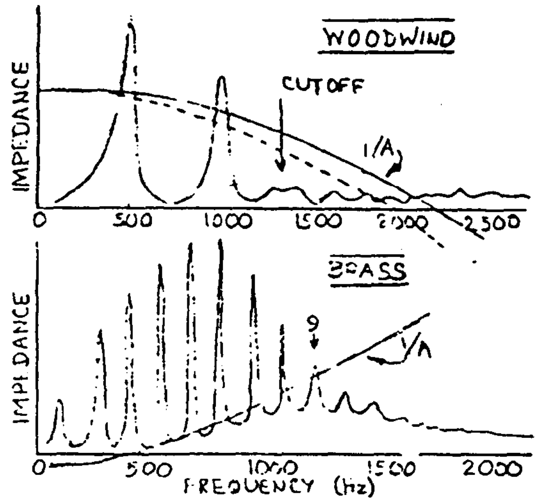
$$(1/A) = (1/A\ )[1 - (\omega/\omega_r)^2] \qquad (4)$$

A similar relationship holds for B, C, D, etc. The fact that 1/A suffers a reversal of sign at ω_r helps us to understand three experimental facts: (a) on woodwinds, it is well above the playing frequency ω_0; (b) on brasses, ω is below ω_0 but always close to it, and (c) in both cases the upper harmonics of the played tone can aid in the oscillation. The upper part of Fig. 1 shows the input impedance curve for a soprano-saxophonelike woodwind fingered to play B , and also shows the curve of 1/A that is typical for a normal (inward-beating) reed. Wherever $Z(\omega)$ is above the 1/A curve, there can be a net production of energy. It is clear that the low-register note is fed by energy inputs at 500 and 1000 Hz, while any higher-frequency components constitute a drain on the system unless the player chooses to put the reed frequency close to 2000 Hz. In every woodwind one finds only a few resonances which lie below a cutoff frequency

f_c determined by the tonehole layout. As a result, energy is produced only within a restricted part of the complete instrumental spectrum.

The lower part of Fig. 1 shows similarly the $Z(\omega)$ and $1/A$ curves for a brass instrument whose player is sounding the note G4. The fundamental component of the note is fed by the interaction of the 3rd resonance peak with the lip reed, which try[?] to resonate at very nearly the same frequency. The player's lips constitute an outward-beating reed so that $1/A_0 < 0$, as is proved by the fact that air column peaks 6 and 9 are observed to contribute energy, despite the fact that they lie above the lip-reed frequency. Once again we find there is a cutoff frequency, this time controlled chiefly by the rate of bell flare.

FIG.1



In both instruments, crescendos and diminuendos can cause pitch shifts as the influence of slightly misaligned cooperating resonances waxes and wanes. Similarly, during the course of a vibrato, one notes significant variations in the oscillatory processes.

It is to be emphasized that the tonehole proportions associated with the entire low-register playing range on a normal woodwind are such as to give a nearly constant value for the cutoff frequency above which the radiation damping kills off any peaks in the input impedance curve. A simplified formula for f is given in Eq. (5) in terms of the bore and tonehole radii a and b, the interhole spacing 2s, and the effective length t_e of the tone hole:

$$f_c = (c/2\pi)(b/a)(2s\ t_e)^{-1/2} \qquad (5)$$

Because f is constant over the whole range, we find that the lowest notes on the instrument are produced by the cooperation of several resonance peaks while the top end of the low-register scale normally arises from only two cooperating peaks. In Fig 1, f_c lies near 1325 Hz.

As we shall see, the value of f_c plays a crucial role in the radiation behavior of every wind instrument and also in our perception of its tone color. For this reason we should notice that the skilled maker has several things--air column shape, mouthpiece proportion, tonehole sizes and spacing, and wall thickness--at his disposal as he seeks to attain proper intermode cooperations for every note, correct tuning, and suitable tone color. What at first seems to be a grossly overconstrained design system actually proves to be quite manageable. Good instruments show a remarkable continuity in their acoustical properties in going from one note to the next, despite the fact that their geometrical proportions may appear irregular to the point of irrationality.

[C7]

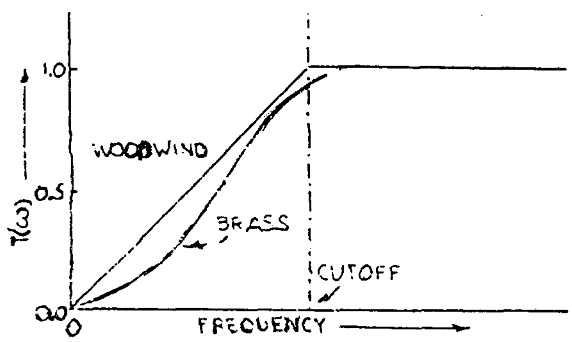
EMISSION OF WIND INSTRUMENT SOUND INTO A ROOM

The internal spectrum measured within the mouthpiece can be related to the room-averaged external spectrum by means of a normalized spectrum transformation function T(ω). This spectrum transformation function is determined by the tone holes and bell of the instrument and is also influenced to some extent by the air-column shape near the mouthpiece end. For air columns with pressure-operated flow controllers, T(ω) has dips and peaks that are roughly reciprocal to those of the input impedance curve, so that emission of sound takes place near to but not coincident with minima of T(ω). It is possible to calculate T(ω) reasonably well for both woodwinds and brasses. For violin-type instruments the analogous function --bridge force to sound pressure--may also be estimated. The analytic form is complicated but its essential behavior at the frequencies making up a tone can be summarized very compactly for conical woodwinds, for the odd components of clarinet tones, and for brass instruments, as in Fig. 2:

> For all the even-numbered components of clarinet tones, T(ω) is roughly unity. The angular distribution of components radiated below cutoff is in all cases isotropic, and above cutoff it becomes progressively more directional along the horn axis (in woodwinds the components radiate in a conical pattern, each with its own half angle).

It is for these reasons that anechoic-chamber wind instrument spectrum measurements made on axis grossly overestimate the relative strengths of the high-frequency components, whereas at 90° from the axis these are greatly underrepresented. Slightly off-axis measurements on woodwinds may exaggerate one or another of the higher partials, while irregularities such as tonehole layout and fork-fingering add greater complexity to the problem, giving sharp nulls or peaks in the amount of a particular component that reaches a particular point of observation.

FIG. 2



There are two reasons why the spectrum transformation function can usefully be defined as indicated. The weaker reason belongs to physics: spectrum measurements made in a reverberant room of sufficient size and with suitable averaging of the analyzed signals from several microphones give a reasonably good correlation with predictions based on the measured internal spectrum, so that one at least has a stable and well-defined physical situation. Of greater importance is the fact that the human auditory system processes two-ear data, extracting signals from the transient and steady-state response of studio-sized rooms to perceive a sound that is quite similar to what one gets from a loudspeaker fed from the internal spectrum via an electronic filter whose characteristic matches T(ω). Thus, an integrating chamber technique that uses several microphones (whose signals must be analyzed separately before averaging) provides data that are physically and perceptually relevant, whereas anechoic chamber measurements can be extremely difficult to interpret in a musical context.

We notice from Fig. 1 and from Eqs. (2) and (3) that the internal spectrum is strongest in its lower components, very little being present above cutoff. On the other hand, the treble-boost action of T(ω) accentuates these weaker high components in the room-averaged spectrum and attenuates the strongly produced lower components. As a result, one is seldom able to deduce the cutoff frequency of a wind instrument from an inspection of the measured external spectrum (except via its angular dependence). The musical situation is quite different: we readily perceive the difference in tone color produced when the cutoff frequency of an instrument is changed by only two or three percent, even when other things are kept the same with regard to reed, mouthpiece, tuning, and alignment of resonances. There is on the other hand a clearly recognizable similarity between the tone color of today's English horn, a baroque oboe, and the single-reed Hungarian tarogato--all of which share a cutoff frequency near 1100 Hz. Similarly, today's alto flute played between its written A4 [?] and C6 easily masquerades as a baroque flute. Throughout the orchestra, darker tone color is associated with lower cutoff frequency. This explains why Beethoven's C clarinets are now replaced by today's B-flat instrument, whose cutoff matches the older C instrument reasonably well, whereas today's C clarinet has a cutoff that is some ten percent higher.

The radiation properties of instruments of the violin family in some respects are reminiscent of those described here for wind instruments. At low frequencies, where the wavelength is long compared with the body dimensions, the radiation is isotropic. At high frequencies the increasing complexity of the plate vibration patterns ultimately leads to a dipole-type pattern with lobes of maximum emission in a direction normal to the plates. This limiting[?] behavior is reached when the autocorrelation length of the plate vibrations is short compared with the wavelength in air. At somewhat lower frequencies the radiation pattern is complex in the manner described for irregular tonehole layouts.

[Note by V. Benade: The sections on Wind Instrument Sound Production and Emission of Sound into a Room were copied from very pale and faded ditto copies. As elsewhere in this set of handout sheets, square brackets enclosing a question mark indicate that something was difficult or impossible to read with any accuracy.]

D. ASSIGNMENT 2

ELEMENTARY TONEHOLE (ETC.) LAYOUT

Read all three--do only one! (Check with me on your choice)

I. Part A. Flute in F, based on a piece of plastic pipe of 17 mm ID

1. Tabulate all the nominal tube lengths for semitones between written C5# and C4.

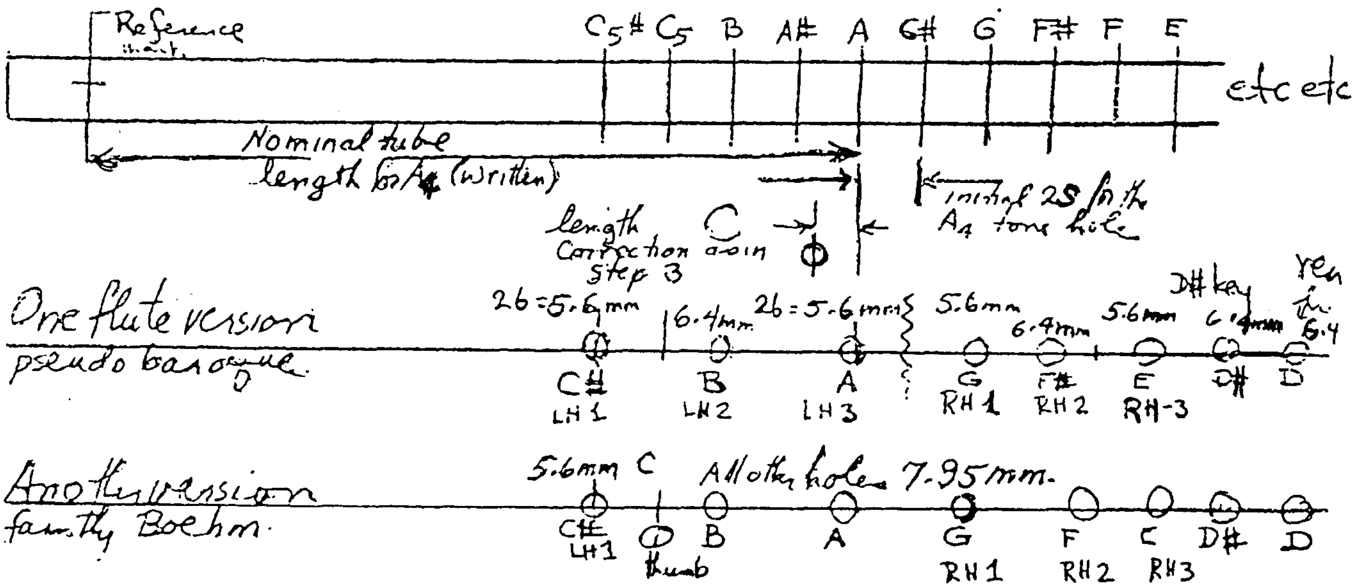
2. Calculate from this listing the initial version of the interhole distances (2s) for the tonehole system under consideration.

3. Using these values of 2s together with the bore diameter and wall thickness (2.2 mm), calculate the tonehole length correction C for each hole.

4. Tabulate the initial tonehole positions by calculating ($L_{nominal}$ - C) for each hole--then subtract to get the new interhole spacings (2s).

5. Use these to calculate new C's for the holes. Relocate the toneholes by using these C's instead of the initial ones as in step 4.

6. Compute the new (2s) values, etc., etc. Iterate until the tonehole positions stay within 1/2 mm as you repeat.



$$C = \left(\frac{2s}{2}\right)\left\{\sqrt{1 + 4\left(\frac{t_e}{2s}\right)\left(\frac{2a}{2b}\right)^2} - 1\right\}$$

$2a = 17\text{mm}$ $\quad t_e \simeq t + 0.75(2b)$ $\quad t = 2.2\text{ mm}$

I. Part B. Figure out how you would decide where to drill the embouchure hole (11- or 12-mm diam)

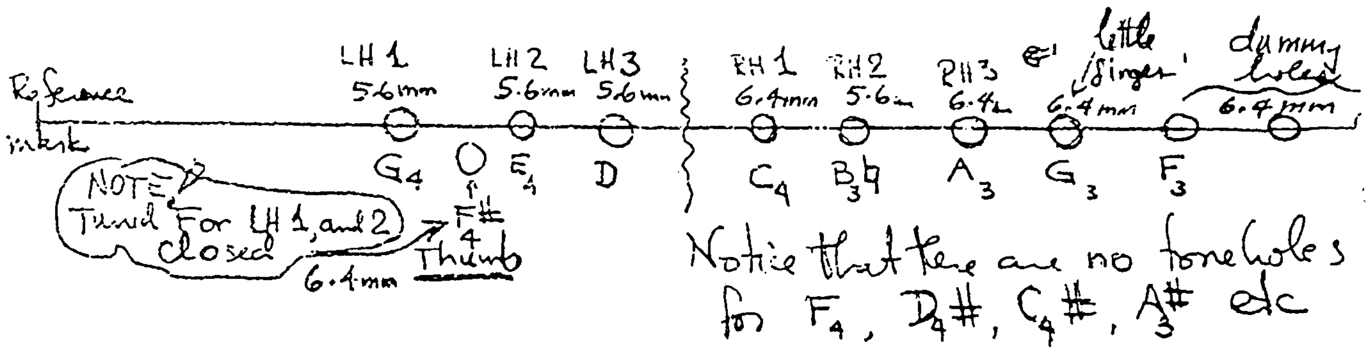
Note: I have tubing, and once your work is checked we will make and test the flute.

II. Part A. Clarinet in B-flat, based on a piece of plastic pipe of 17 mm ID and 2.2 mm wall thickness.

This problem is identical in form to section I on the flute, except that the pseudo-real instrument has a different arrangement of open holes and missing holes. WE WILL NOT BOTHER TO EXTEND THE SCALE ACROSS THE BREAK.

1. Tabulate all the nominal tube lengths for semitones between written G4 and D3.

2, 3, 4, etc., etc., as before.



II. Part B. Figure out how you would decide where to attach the mouthpiece (a standard one).

Note: I have tubing, and once your work is checked we will make and test the clarinet.

III. Part A. Consider a baroque D trumpet (related to A = 415 Hz, a semitone below today's tuning), which we will take initially to be a Bessel horn for which m = 0.55.

1. Choose a length L (← 2.33 meter) which puts mode 4 in tune with the note to be played (let y_0 = 30 mm).

2. Calculate the frequencies of modes 2, 3, 5, 10 and 12 for this horn, and from these calculate the discrepancies D = (f_{calc} - $f_{desired}$)/$f_{desired}$ for each of these modes (note that we have chosen f such as to make D4 = 0). Here $f_{desired}$ is the modal frequency we want for good tuning and good intermode cooperation (use just tuning!).

3. Study FMA, Fig. 20.12, and accompanying text, then sketch out a good freehand curve for L vs. frequency for a mouthpiece for which the popping

frequency is 750 Hz and total volume is 4.55 mm³--take the trumpet bore diameter to be 12 mm.

4. Read off from this L_{eq} curve the L_{eq} corresponding to each of the desired modal frequencies. Prepare a set of ΔL's from these by subtracting the mode-4 L_{eq} from each of the others.

5. The fractional shift $(\Delta f/f)_n$ of any modal frequency above mode 2 of a usable brass instrument may be calculated from any corresponding equivalent length change ΔL by use of the formula $(\Delta f/f)_n = [(\Delta L)/(2c)] f_4$ based on the frequency of mode 4 of the instrument (c = speed of sound). Use this relation with the mouthpiece ΔL's from step 4 to see how closely the mouthpiece does the job of pulling the Bessel horn frequencies into line.

III. Part B. Suggest reproportioning of the mouthpiece that will improve the situation uncovered in step 5.

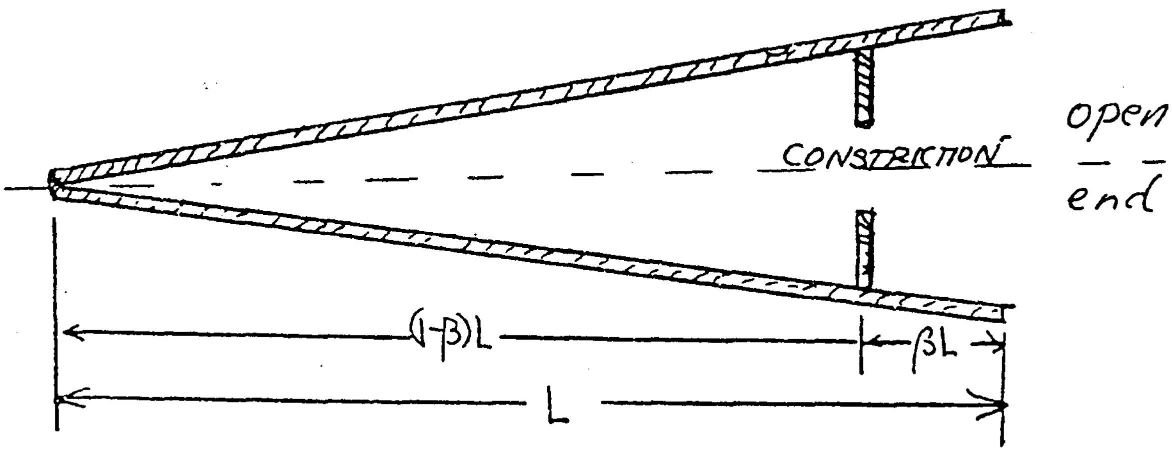
Note: We'll make and test some mouthpieces on a baroque trumpet.

added by III, Part A, 2:

f_2	-92¢
3	-31
4	000
5	18
6	31
8	45
10	54
12	60¢

F. ACOUSTICAL BASIS OF HAND-STOPPING OF HORNS [note that there is no section E]

Consider a horn that is a complete cone, with a constriction inserted part way into it at a distance βL in from the big end (the horn has an overall length L).



Let H be a measure of the strength of the constriction.

H = 0 ⟶ H = intermediate ⟶ H = ∞

Complete and open-ended horn $k_nL = n\pi$ gives natural frequencies

$$f_n = \frac{nc}{2L}$$

Shortened cone, closed completely at length $(1 - \beta)L$

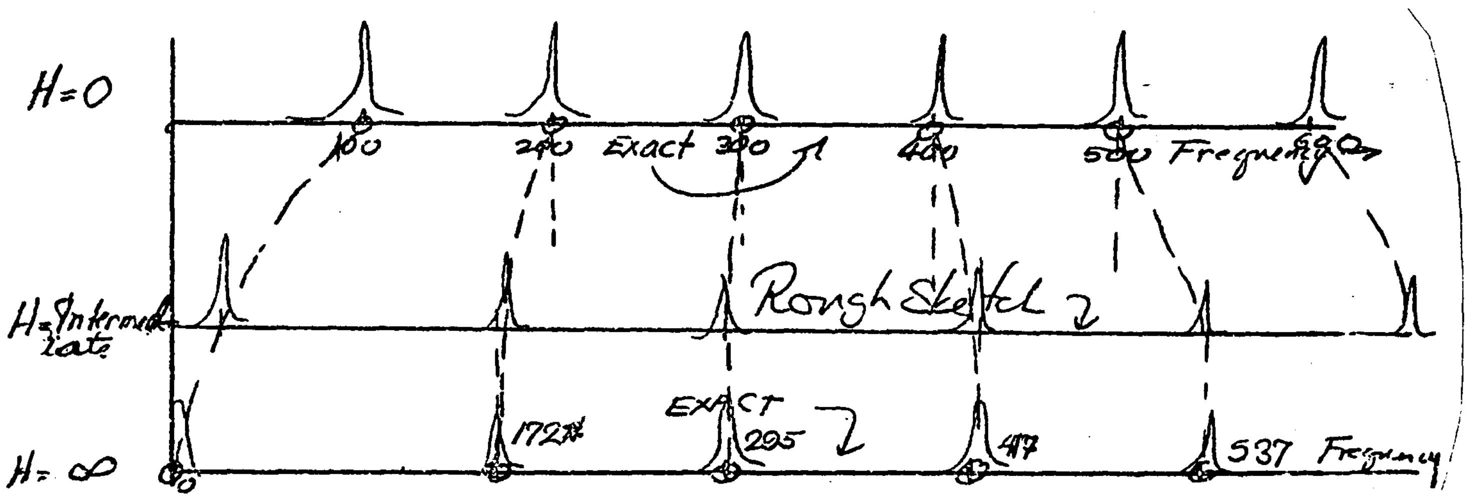
$$k_n(1 - \beta)L = \text{Ti} \times \begin{cases} 0.0 \\ 1.430 \\ 2.459 \\ 3.471 \\ 4.477 \end{cases}$$

$$f_n = \frac{c}{2(1 - \beta)L} \times \text{one of these}$$

(The calculation is a bit complicated--if you are interested, ask me about how it is done, for every arbitrary value of H.)

[F2]

LET US MAKE A FREQUENCY DIAGRAM FOR THE CASE WHERE β = 0.1667



NOTE! INCREASING H (a) moves lower-numbered peaks to lower frequencies.
(b) moves higher-numbered peaks to higher frequencies.

Let us see what this means with the help of a "practical" example. Suppose we want to flatten the 400-Hz note based on peaks 4, 8, 12, . . . of the complete horn. The hope is to find something that plays about a semitone down, around 0.96 x 400 = 384 Hz. Its harmonics are at 768, 1152, 1536 Hz.

<u>First trial</u>. Assume H = 4. Numerical calculation shows:

<u>Fund.comp.</u> Mode 4 at 378.4, with harmonics at 756.8, 1135, 1513 Hz

<u>None at 2nd</u> Mode 7 is at 688.8, too low by far to feed 2nd harmonic
Mode 8 is at 792.8, considerably too high to feed 2nd harmonic

<u>3rd some</u> Mode 11 at 1183.9 Hz, a little high for feeding 3rd harmonic

<u>4th some</u> Mode 15 at 1528.6 Hz, only little high for feeding 4th harmonic

<u>Second trial</u>. Assume H = 7 and do the calculation to find:

Mode 4 is at 375 Hz, with harmonics at 750.4, 1125.6, 1500.8 Hz

Mode 7 is at 688 Hz, which straddles the desired 2nd harmonic

Mode 8 is at 791 Hz, " " " " " "

Mode 11 at 111.6 Hz, too low for 3rd harmonic, only <u>by a little</u>

Mode 12 is at 1212.5

Mode 15 is at 1326.8, a little high for the 4th harmonic

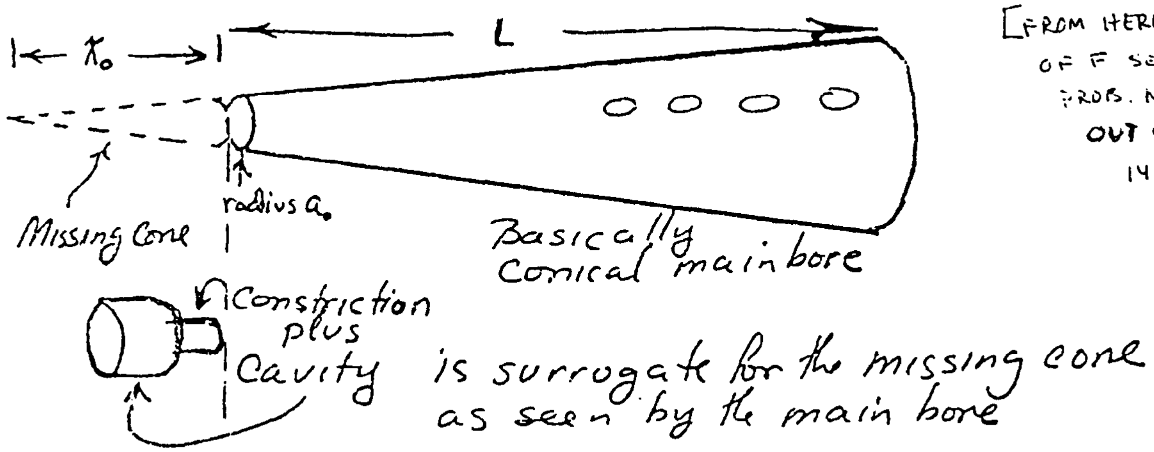
Third trial. We notice that some value of H lying between 4 and 7 will give a horn on which the 4th mode cooperates well with the 11th and 15th modes to feed the 1st, 3rd, and 4th harmonics of the tone.

Look at the hand-horn resonance curve of FMA, Fig. 20.17 to see how the peak heights will work out with these ideas.

Remember that the lip reed feeds energy above, not below, its own resonance frequency, but the break-even line rises at high frequencies. What has this to say about hand-horn techniques for the upper notes??

BASIC ACOUSTICAL RESOURCES FOR ADJUSTMENT OF CONE-PLUS-REED COMBINATION

[FROM HERE TO END OF F SECTION WAS PROB. NOT HANDED OUT UNTIL LECTURE 14, OCT 3, 1977]



(1) at very low frequencies (i.e., near fundamental component of lowest note on the instrument) the air column is fooled if the total volume (cavity plus constriction) is pretty much equal to $V_o = (1/3)\pi Q_o^2 x_o$ --the volume of the missing cone.

Note: The effective volume includes effects produced by springiness of the reed cavity wall plus some associated with the dynamics of the oscillation process. See FMA, sec. 22.1, Fig. 22.1, pp. 465 and 466.

CAVITY: it is within the oboe or bassoon reed or sax mouthpiece.
CONSTRICTION: it is the oboe reed staple, bassoon bocal, sax neck.

(2) At frequencies that are fairly high (e.g., 2nd-mode frequency of shortest used tube), there is a resonant frequency of the cavity plus constriction [the playing frequency of this object using "standard enbouchure" (!?!)]. Call this f_{rs}. Now, f_{rs} must pretty well match the natural frequency of the missing cone segment.

$$f_{rs} \cong \frac{c}{2x_0}$$

Don't mix up f_{rs} --the played frequency of reed on its own cavity and neck

f_r (deducible via cooperative effects)--the natural frequency of the cane by itself as held in the player's embouchure.

(3) The general behavior of the air column in response to changes in the $V_{effective}$ and the f_{rs} :

(a) Enlarging the total effective volume lowers mode 1 some, mode 2 more, and mode 3 a lot--it NARROWS FREQUENCY RATIOS--this is in cents or percent, not in Hz!

(b) Lowering f_{rs} lowers the frequencies of the air column modes that lie in its neighborhood.

(c) Pinching with lips or blowing harder collapses cavity, which reduces V_{eff} and raises f_{rs}. NOT BIG EFFECT ON SAX.

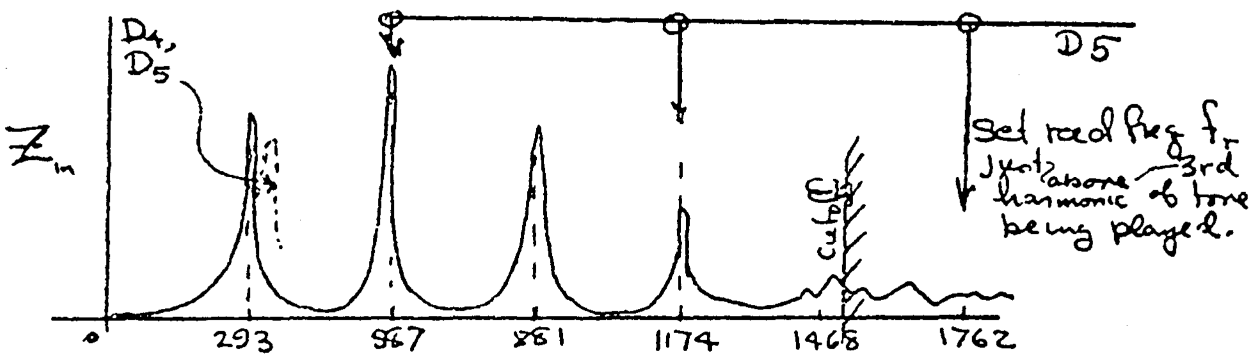
(4) The reed's own natural frequency f_r also rises with increased lip or wind pressure.

G.

THE FORMAL WAY TO MEET AN OBOE

FORGET ABOUT TUNING FIRST--MAKE IT SING--

(1) Set embouchure to get best centering and singing quality on D5 (the second D above middle C).

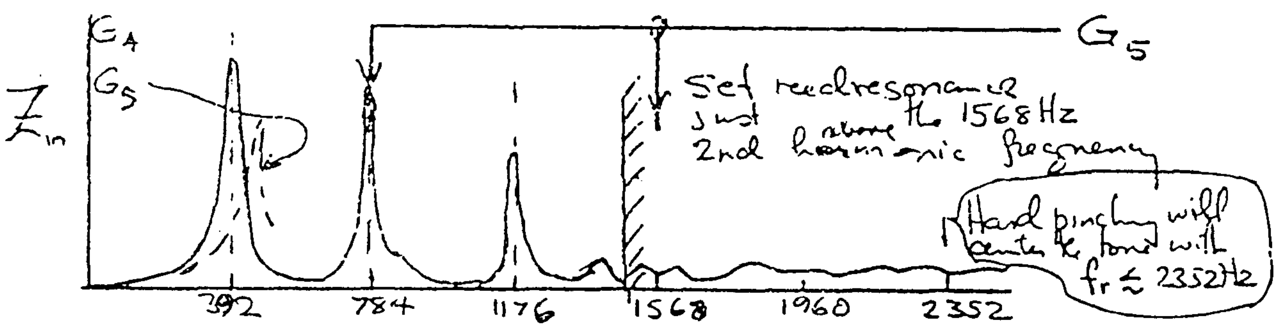


This is almost always a very stable, easily set note. Peak 4 is not very tall and the reed frequency is comfortably in the range of easy settability. Note: a very tight embouchure will center a sharp-pitched sound with f_r feeding the 4th harmonic of the tone, at 2349 Hz.

(2) Check your setting by closing the half hole--the oboe should stay in the 2nd register, or gradually drift down an octave. It should come down neatly if the embouchure is SLIGHTLY slackened to put f_r below 1762 Hz.

(3) Re-obtain the magic embouchure setting of step 1 or take one that is a trifle more slack; when this has been done, close the register hole and play the low D4. If the oboe and the reed-plus-staple are OK, the octave will be accurate. A narrow octave implies too large a total effective volume of the reed and staple. Confirm by pinching the reed [with your fingers] while playing--either across the bulge to reduce the volume, or at the edges, which increases it. ↓⊖↑ or →⊂⊃← The tone will center best when the volume is right.

(4) Reset the D5 embouchure once again, then play G5 above it. A slight slackening of the embouchure for blowing pressure will lower the reed resonance to where it will feed the second harmonic of the tone.



Side remark: What happens if one plays chromatically up from D5 to G5?!?

(5) Reset everything via D5 and G5 as before. Check that closing the register hole pretty much keeps the instrument in the second register. Again, a slight slackening of the embouchure will let the tone drop to G4 as the reed resonance drops below 1568 Hz.

(6) If the G4 and G5 are an accurate octave apart, all is well. Sometimes if the oboe and reed are not quite right, G4 centers with a different embouchure than does G5--one even finds <u>two or more</u> embouchures that center the G4 (each with its own pitch). The reasons are as follows:

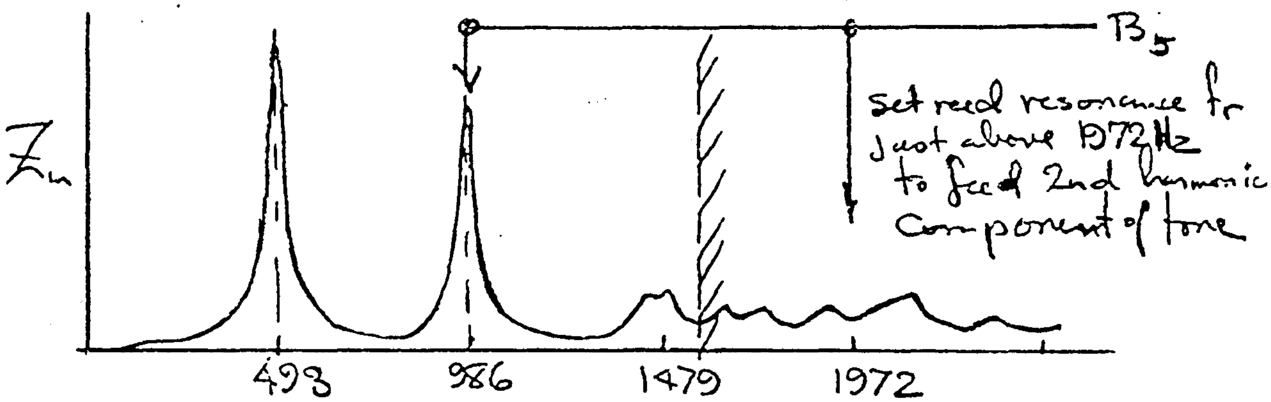
(a) One aligns peaks 1 and 2 by changing the net reed volume.

(b) One aligns the reed resonance around 1568 Hz with four times the peak-1 frequency so that it nicely feeds the 4th harmonic of the G4 tone.

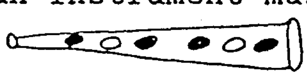
(c) One aligns peak 3 to feed the 3rd harmonic of an oscillation whose fundamental lives off peak 1.

Etc., etc. It's nice when one gets it all together!

(8) Reset at D5 and then finger B5 using the "short" fingering. Once again center the tone with proper adjustment of f_r.



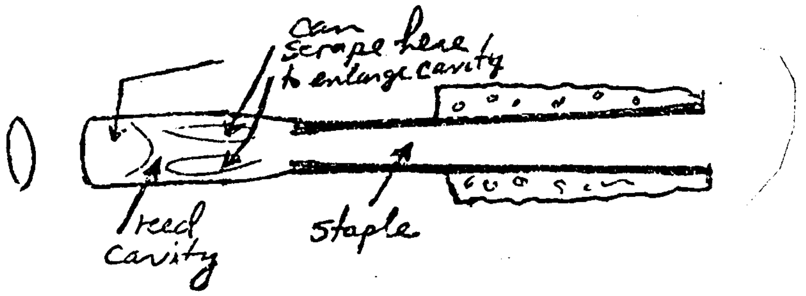
(9) It will be possible to keep the second-register note going sans register key if the reed resonance is properly placed (except on the older oboes--e.g., baroque--where the low tonehole cutoff frequency makes peak 2 less tall than usual. Such an instrument must resort to a suitable "long" fingering to play B at all:



(10) Slightly slacken the embouchure and let B5 drop to B4. If the octave is good, all is well, and the lower note will be centered via good cooperation between peaks 1 and 2 plus some nourishment of the 4th harmonic via the reed resonances (if one retightens the embouchure to put f_r slightly above 1972 Hz).

(11) If the octave sneak is narrow between B5 and B4 as outlined above, while the total reed-plus-staple volume is correct (as checked in step 3 above), it means that the playing frequency f_{rs} of the reed plus staple using "normal embouchure" for the note is too low. Normal embouchure here means the one that centers when playing B5 as described in steps 8 and 9. [An oboe player's empirical method for getting f_{rs} right--"I sing a "nothing" embouchure as when making and checking reeds. The reed should "crow" at C4 with a possible C5 harmonoic evident. Some players try for a 3-octave crow, but most don't care. If the reed crows B4, the top of second octave usually sags in pitch."]

(12) A little about tinkering the total volume and f_{rs} for a reed:



Elongating, widening, or thinning the cane enlarges V_{total} and lowers f_{rs}. Enlarging or shortening the staple enlarges V_{total} and raises f_{rs}.

So far we have been confining our attention to the job of making the oboe sing, note by note, without worrying about the tuning of these notes into some semblance of a musical scale. The octave checks were used only as a diagnostic tool to find out about the relation of reed to air column for the job of producing a nice, steady oscillation.

> On a properly made oboe of any vintage, making it sing will also put it very well in tune with its own scale, at its own reference pitch (e.g., equal temperament based on A-440). To be sure, certain notes may be sharp or flat as a part of the working compromises for a particular system of tone holes and keywork, but you are expected to know about and understand such features as a matter of course.

Let us look at the whole question of sorting out the oboe from the point of view of tuning, rather than of tone and response. This is a very slippery business, but sometimes one must go through something of the sort to correct one or two notes that may not be right because of makers' errors or later tinkering.

(A) Suppose the D4-D5 octave is OK, but the scale runs flat in going up from D4 to C5 (i.e., the low-register scale runs short with an extra big jump between C5 and the D5 that is the bottom note of the second register. If everything sings as described earlier, then try to renegotiate with a narrower staple and/or a narrower reed--but you must keep aware of other tuning checks given below, so as not to get into a bind! (Shorter should somewhat do the job too, but save it for what it does best--see below.)

(B) Suppose the D4-D5 octave is OK, more or less, but the whole scale seems a trifle flat from what you want it to be. THIS IS A DANGEROUS GAME IF YOU DON'T KNOW FOR A FACT WHAT PITCH THE INSTRUMENT WAS DESIGNED FOR!

Provided the instrument pretty well sings with the reed you have and provided the scale runs a trifle short in the manner indicated in step A, the cure is to use a slightly SHORTER staple and/or reed--a narrower one of each will stretch the scale rather than raise it en masse. THIS SEEMS OBVIOUS--IT IS NOT, HOWEVER. It is a special quirk of a cone of variable length provided at the top with a cavity--somewhat elongated--and a constriction.

As little as 3 mm shortening will raise the overall playing pitch of essentially the whole scale by 15 to 20 cents on an oboe if the cooperations are in reasonably good order.

THIS IS NOT TRUE FOR THE FLUTE (conical bore or cylindrical bore) OR THE CLARINET.

(C) If you have to raise the pitch as in step B or stretch the scale as in A, the cooperations may have been upset, and you have to go through the whole business outlined in the previous parts of this section on the oboe.

ACTUALLY, ONE KEEPS ALL OF THIS MATERIAL IN MIND AT THE SAME TIME WHILE NEGOTIATING THE PROPER PROPORTIONS FOR REED AND STAPLE.

EVERYTHING IN THESE NOTES TRANSLATES WITHOUT ESSENTIAL CHANGE FOR THE BASSOON;

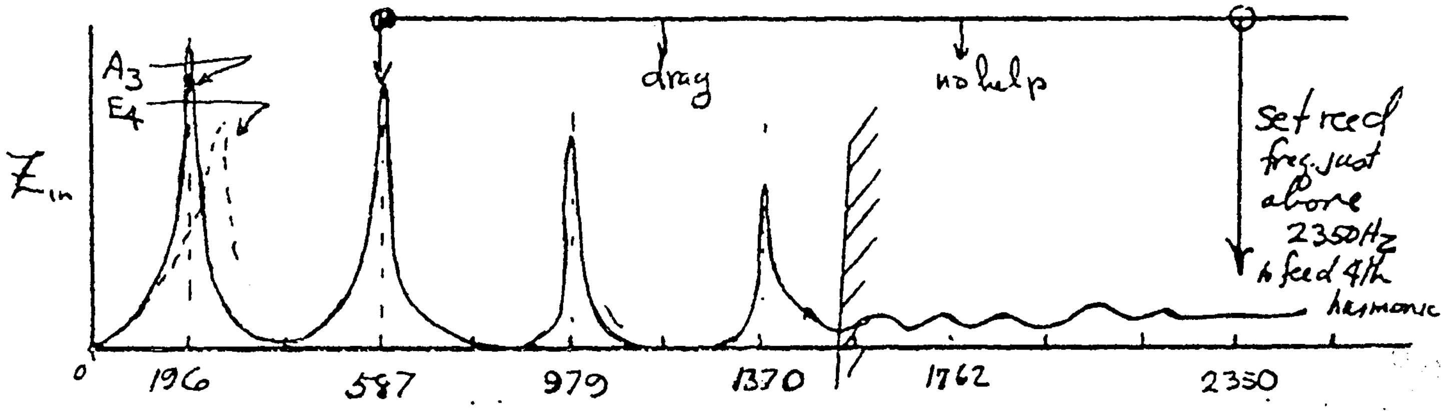
WITH ONLY A LITTLE CHANGE IT ADAPTS THINGS TO THE SAXOPHONE;

AND IT IS NO GOOD AT ALL FOR FLUTE OR CLARINET.

H. TO MEET A CLARINET

Let us run through the procedure for a B-flat clarinet, with the understanding that the other members of the family show basically similar behavior. (Departures for other members of the group include the following: the big clarinets give their players less range of adjustment of reed frequency; the A clarinet behaves a little differently because the same reed is used on a lower-pitched instrument that has a lower cutoff.)

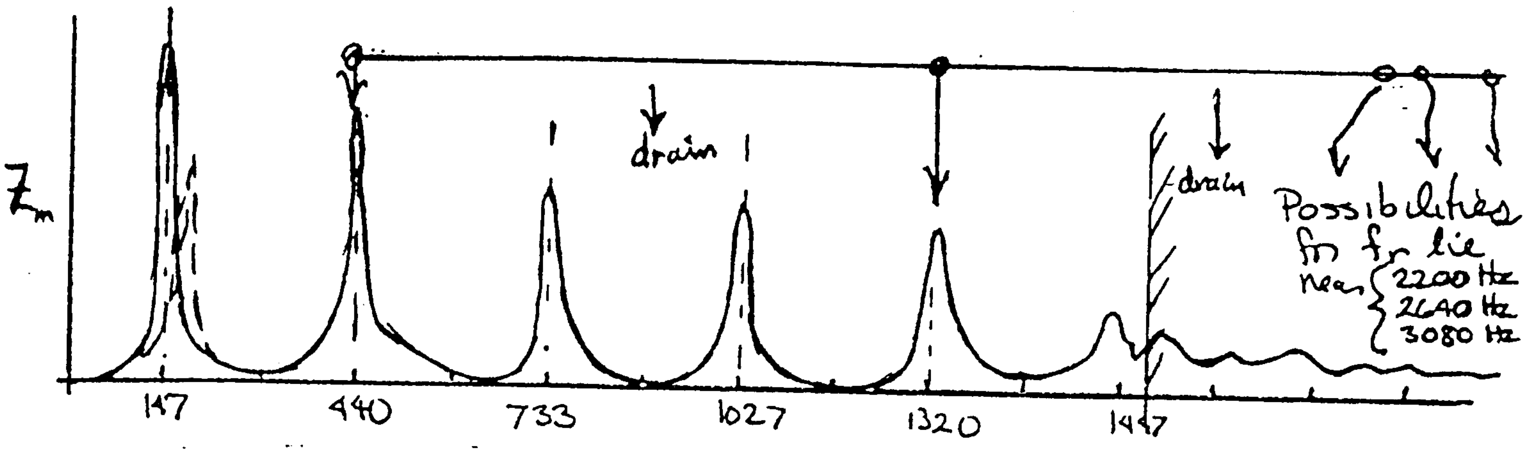
(1) Play E5 and center up the tone. It turns out that this has a unique setting.



There is only one way the reed resonance can be gotten in line with the harmonics of this note. 1762 Hz, to feed the 3rd harmonic, is too low to reach without severe strain. The 5th harmonic at 2935 is reachable (just barely), but it can't keep the oscillation alive without the register hole since it is too far from the source of the fundamental's energy. (Energy will drain at harmonics 2, 3, and 4 under these conditions.)

(2) Make sure you are properly set by ascertaining that the tone persists at mf level with the register hole closed, but will restart at low A3 if the reed is touched with the tongue. A slight slacking of the embouchure from proper setting will also produce a drop to A3. Question: Why does a too tight embouchure also cause a drop?? IMPORTANT!

(3) Play quickly down the scale from a properly centered E5 to B4, slacking your embouchure as you go. (Do not try to maintain the intermediate notes sans register hole as yet!) Blowing at a mf to f level, find the centered tone that is confirmed by your ability to maintain the B5 without the register hole.



This tone is supported by peaks 1 and 5 (which feeds the third harmonic) and the possibility of reed resonance feeding the 5th harmonic at 2200 Hz
6th harmonic at 2640 Hz
7th harmonic at 3080 Hz

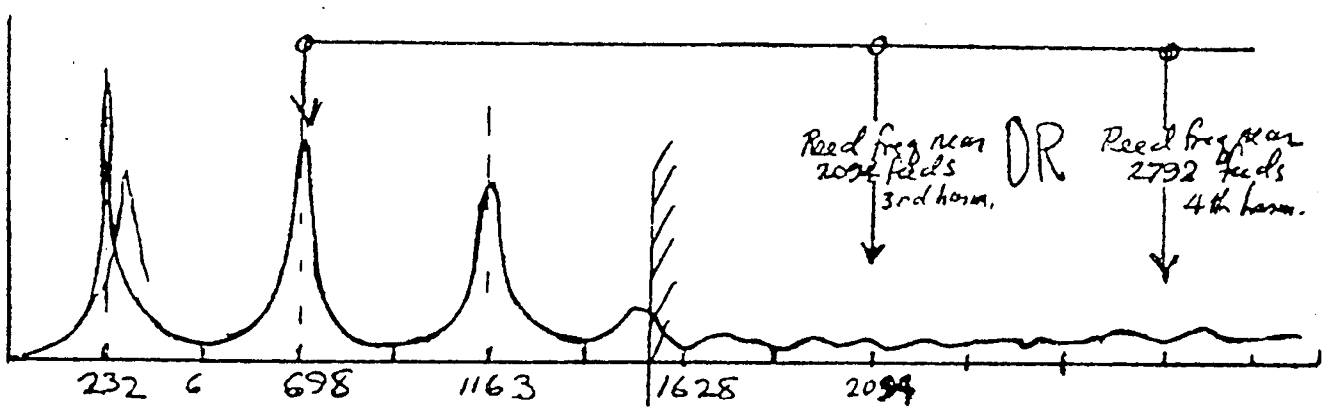
Following the procedure at the bottom of the previous page, you will have set f_r just above 2200 Hz.

Note that the B♮4 tone can center and sing at a higher and at a lower pitch! This is musically important.

(4) If you center the E5 again, as in step 1, and then tighten (or slack off) enough to repeat the performance at D5, your reed frequency will be near 2640, which feeds the 5th harmonic of D5. A sudden closing of the holes to finger B♮4 again will show it is possible at forte playing levels to keep B♮4 going without register hole--this by way of reed resonance help at the 6th harmonic. The pitch of this B♮4 is a little higher than that obtained in step 3.

(5) Pinching hard will sort of center the B5 by the harmonic input from a reed tuned just above 3080 Hz. But one cannot usually keep the sound going without the register hole.

(6) Return to E5 and reset your embouchure as in step 1. Now play up the scale with slowly tightening embouchure to G5.



[H 3]

This procedure will leave the embouchure set to give $f_r \approx 2792$ H, which can be centered up to feed the 4th harmonic of the G5 tone. It is possible to keep the sound going without the register hole. A more centered and clarinetish (odd harmonics strong) sound can be gotten by playing a little flat to bring f_r down to feed the 3rd harmonic at 2094 Hz.

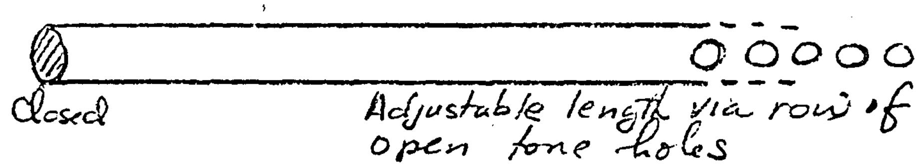
This f_r is getting too low for real comfort

(7) Notice that f_r set at about 2792 to feed the 4th harmonic of G5 is also near a setting that will feed the 3rd harmonic of the C5 above, or the 5th harmonic of the D5# below. It is worth some careful thought about the implications of this remark, and of step 4 above, for the tuning of a clarinet by its maker and for the intonation behavior of the instrument in the player's hands.

(8) A very similar procedure to all the above shows that B5 is best fed with f_r set to feed the 2640-Hz 3rd harmonic. We recognize that the same embouchure setting will very nearly center up F5# via input to its 2640-Hz 4th harmonic or the 5th harmonic of D5. Reconsider step 7 above!!

It is very instructive to follow through and to observe by playing experiments the behavior of other parts of the scale, recalling that $2000 \leq f_r \leq 3000$ without strain.

We will use the clarinet as our initial prototype.

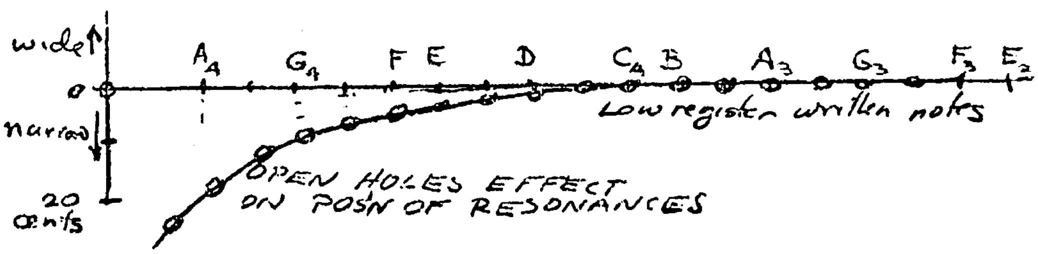


(1) (a) $C = s[\sqrt{1 + 2(t_e/s)(a/b)^2} - 1]$ at low frequencies--rises as one gets near cutoff.

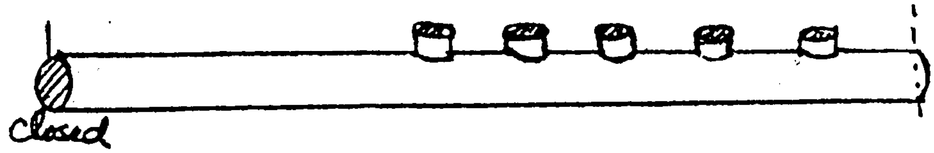
(b) $f_c \quad (c/2\pi)(b/a)\sqrt{1/(2s \cdot t_e)}$ very important for tone color, etc.

This has to do with <u>open</u> tone holes. A cylindrical pipe plus a row of open holes will have its higher-frequency resonances at slightly low frequencies as a result of the fact that C calculated at low frequencies is too small at high.

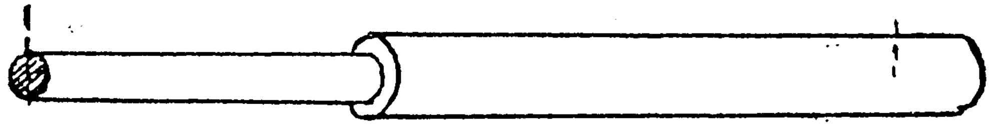
ON AN INSTRUMENT WITH FIXED f_c OVER THE SCALE, the ratio f_2/f_1 runs short of the desired 3/1 ratio at the NORTH end of the air column.



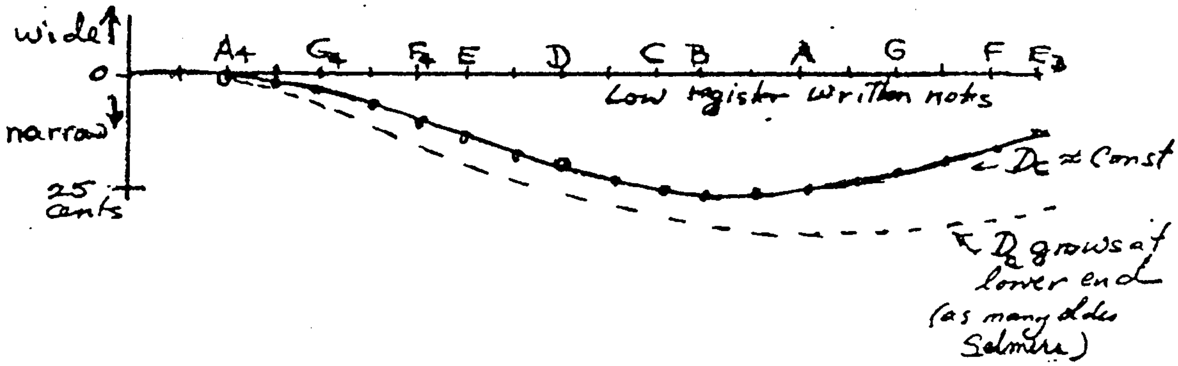
(2) Look at a cylindrical pipe provided with a row of <u>closed</u> tone holes:



The closed-hole bore acts enlarged (lower Z_c) and elongated (lower V_{phase}) (see next section, Math. of Waves, parts 1 and 2, for the math of this):



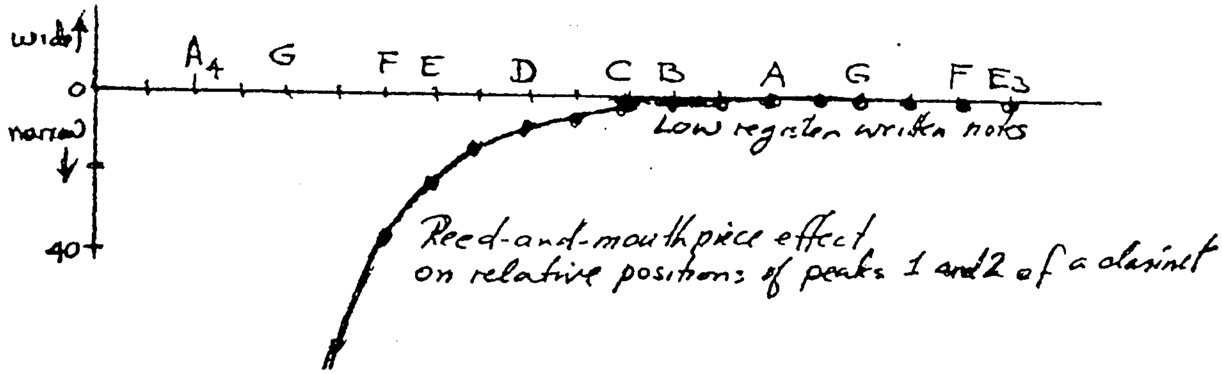
Effective cross section is $\pi a^2[1 + (1/2D_c)]$
$D_c = (b/a)^2(L/2s) = 0.08$ is a good, round number
Elongation is by the same $[1 + (1/2D_c]$ factor
Notice that this enlargement of the lower bore narrows the f_2/f_1 ratio, if D_c = const along the bore (which is roughly what one finds),



The maker must also progressively move tone holes NORTH as we go down the scale to compensate the "elongation" effect.

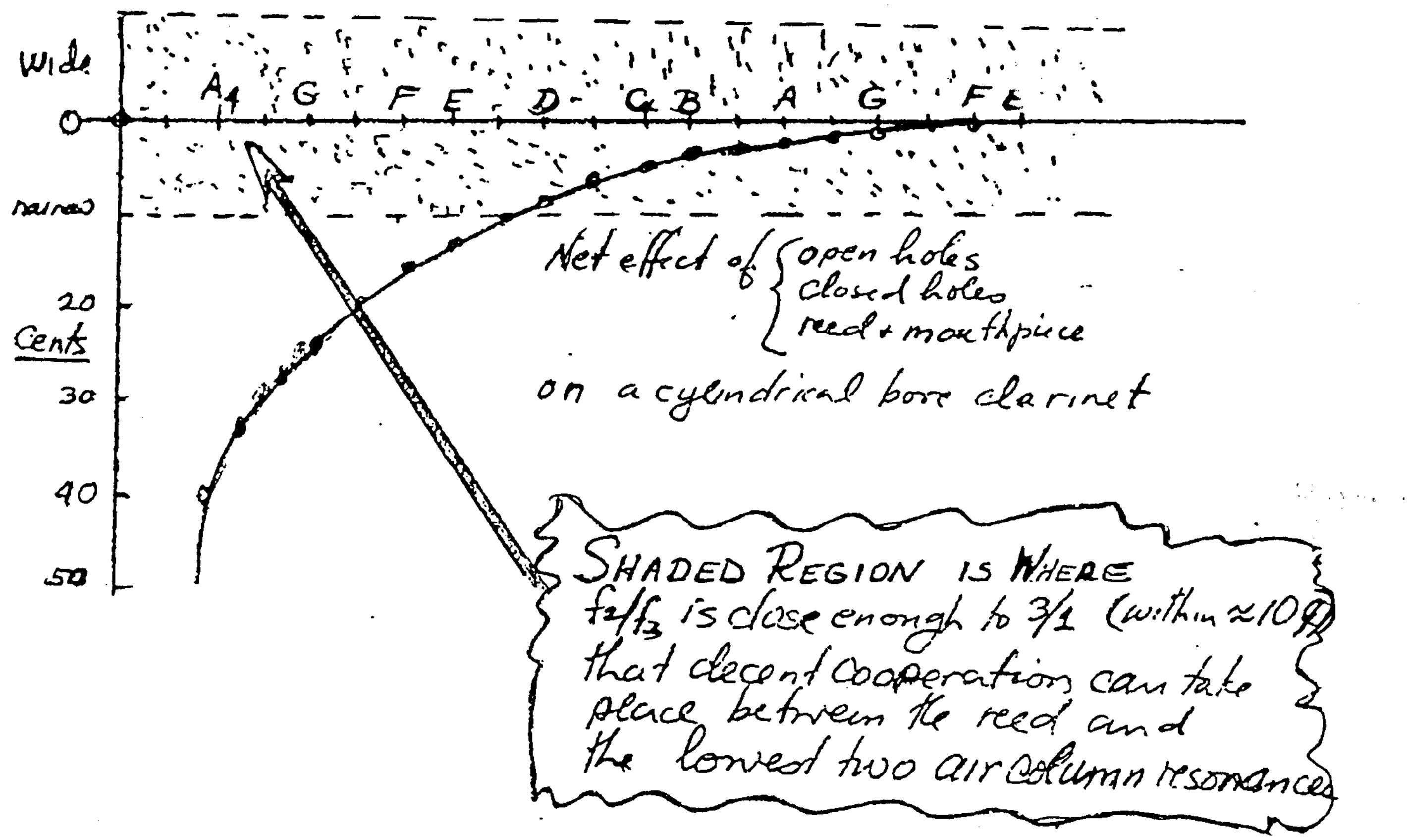
<u>NOTE</u>: ALL THIS IN THIS SECTION is based on detailed calculations (mostly done before 1968), measurements on many instruments, and experience in adjusting numerous instruments. I will present specific examples next week.

(3) The reed-and-mouthpiece equivalent volume is not quite constant over the frequency range (see FMA, sec. 22.2). This also serves to narrow the f_2/f_1 mode frequency ratio, in an amount that is greater at the throat-tones end of the instrument than it is lower down.



(4) There is an overall and CONSISTENT widening of the f_2/f_1 ratio across the entire scale because of a frequency dependence in the frictional and thermal effects arising at the pipe walls. This amounts to about 10 cents.

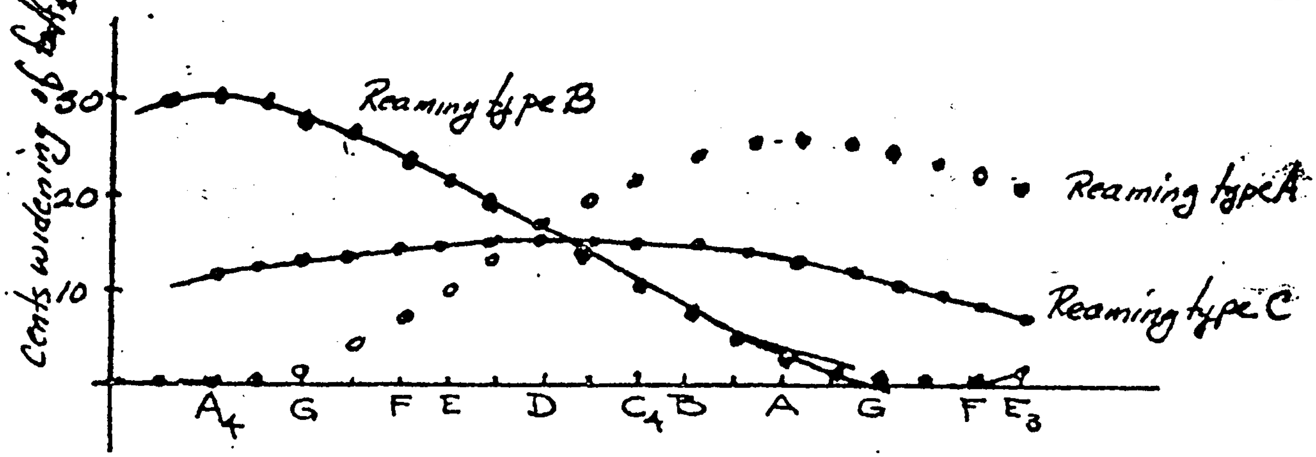
(5) Let us pull all this together to see what happens to any <u>entirely</u> cylindrical clarinet tube provided with normal tone holes and a normal mouthpiece. AGGREGATE ALL OF THE ABOVE:

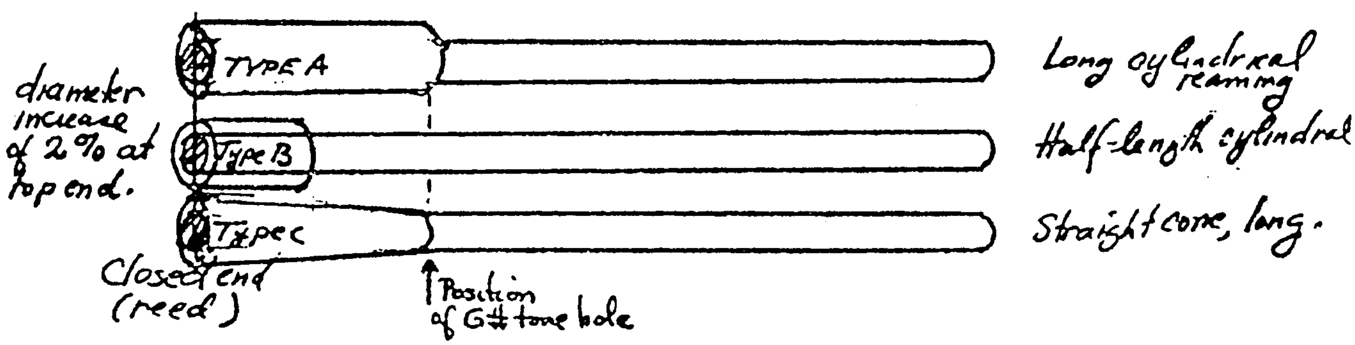


This clarinet would have ABOMINABLE throat tones, a poor G4, a tolerable (familiar) F4, and superb notes to the bottom of the scale beginning from D4.

(6) It is time now to ream the upper bore to fix the effect described in step 2, along with anything else that can be helped. I find it very useful to work from computed curves like this:

[H6]





HOW TO USE THIS?

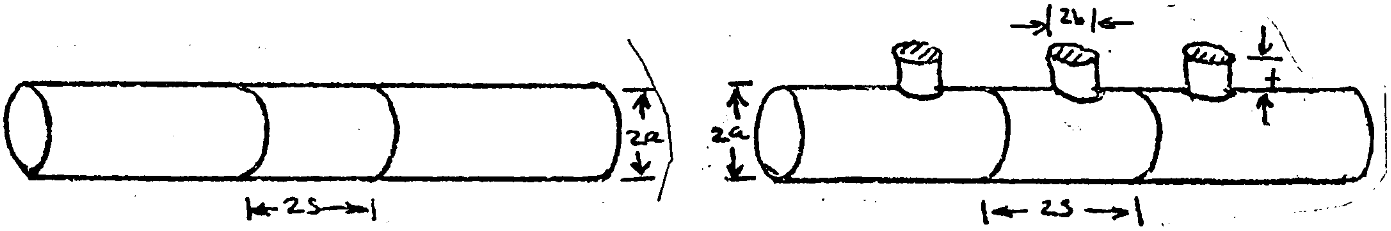
Notice that a full dose of type-A reaming will correct the problems of step 2 if this were present by itself, but would merely louse up the overall clarinet as sketched in step 5. However, an 80% dose of type B would pretty well fix up the works up as far as F4# or so. AS THE CLARINET STANDS, THESE CURVES WILL NOT DO THE FULL JOB.

A heavy dose of type-D reaming, half-length cone, might help, BUT CLARINETS ARE NOT MADE WITH CYLINDRICAL BORES CLEAR TO THE BOTTOM (which we then enlarge in the upper parts).

SO FAR WE HAVE ONLY WORRIED ABOUT THE LOW-REGISTER COOPERATIONS. LOOK NEXT AT WHAT HAPPENS IN THE CLARION (SECOND) REGISTER. Tune in next week, folks!

I. MATHEMATICS OF WAVES IN A PIPE WITH CLOSED HOLES

We will compare, in parallel columns, the derivation of the wave equation for a pipe without, and with, closed toneholes. [See AHB, On the math. theory of w.w. finger holes, JASA 32 (1960), for entirely diff. derivation.]



typical section--
short compared
with wavelength

typical section--
short compared
with wavelength

The left-hand end of our typical section will be located at x. The right-hand end is then at x + 2s. Let the longitudinal displacement of the air at the left end at some time be y(x,t). Then the displacement at the right is $y(x + 2s,t) = y(x,t) + (\partial y/\partial x)2s$.

NEWTON'S 2ND LAW

(1) $$[\rho(\pi a^2)2s]\ddot{y} = -(\pi a^2)\frac{\partial p}{\partial x}2s$$

moving mass — pressure diff

For both, since air in closed hole cannot accelerate ⇄

Tidy it up a little

(1a) $$\ddot{y}\rho = -\frac{\partial p}{\partial x}$$

Volume of air in segment

$$V = (\pi a^2)2s$$

← smooth pipe

Volume of air in segment

$$V_c = \pi a^2 2s + \pi b^2 t$$

$$= \pi a^2 2s\left[1 + \left(\frac{b}{a}\right)^2\frac{t}{2s}\right]$$

Pipe with closed holes →

$$V_c = \pi a^2 2s[1 + D_c]$$

If left side moves differently from right, net volume change in both cases is given by

$$\Delta V = \pi a^2\{y(x+2s) - y(x)\}$$

Both pipes →

$$\Delta V = (\pi a^2)\frac{\partial y}{\partial x}2s$$

In both cases the pressure p produced by a volume change is $-B\frac{\Delta V}{V}$

(2) $$p = -B\frac{\partial y}{\partial x}$$

(2) $$p_c = -B\left[\frac{1}{1+D_c}\right]\frac{\partial y}{\partial x}$$

We need next to eliminate y between Eq 1a and each version of Eq 2. To do this we first take the gradient of Eq 1a

(3) $$\rho\frac{\partial}{\partial x}\ddot{y} = -\frac{\partial^2 p}{\partial x^2}$$

Now differentiate each form of Eq 2 twice with respect to time

(4) $$\ddot{p} = -B\frac{\partial \ddot{y}}{\partial x}$$

(4') $$\ddot{p}_c = -\left[\frac{B}{1+D_c}\right]\frac{\partial \ddot{y}}{\partial x}$$

Eliminate the mixed derivative $\frac{\partial \ddot{y}}{\partial x}$ between Eq 3 and 4

(5) $$\frac{\rho}{B}\ddot{p} = \frac{\partial^2 p}{\partial x^2}$$

(5') $$\left(\frac{\rho}{B}\right)(1+D_c)\ddot{p} = \frac{\partial^2 p}{\partial x^2}$$

wave velocity $=\sqrt{\frac{B}{\rho}} \equiv c$

wave velocity $= \sqrt{\frac{B}{\rho}}\cdot\sqrt{1+D_c} \doteq c(1+\frac{1}{2}D_c)$

This is one of the two properties of the medium

Now For The Wave Impedance: (pressure/volume flow)

Suppose we have a wave $p = p_0 \cos[kx - \omega t]$ ($k \equiv \frac{\omega}{V_{ph}}$)

The velocity of flow is $\pi a^2 \dot{y}$ which we can get by integrating Eq 1 with respect to time

(6) $$\text{flow} = (\pi a^2)\frac{1}{\rho}\int dt\left(-\frac{\partial p}{\partial x}\right) = \pi a^2\frac{1}{\rho}\int dt\left[p_0 k \sin(kx-\omega t)\right]$$

$$= \frac{\pi a^2}{\rho}\frac{k}{\omega}p_0\cos[kx-\omega t] = \frac{\pi a^2}{\rho}\frac{k}{\omega}p(x,t)$$

Recall that $V_{phase} = \omega/k$ so

$$Z = \frac{p_0\cos[kx-\omega t]}{p_0\cos[kx-\omega t]}\cdot\frac{\rho c}{\pi a^2}$$

$$Z_c = \frac{p_0\cos(kx-\omega t)}{p_0\cos(kx-\omega t)}\frac{\rho c}{\pi a^2[1+\frac{1}{2}D_c]}$$

In both cases ~~Z~~ independent of time and position.

(7) $Z = \rho c / \pi a^2$

(7) $Z_c = \dfrac{\rho c}{\pi a^2 [1 + \frac{1}{2} D_c]}$

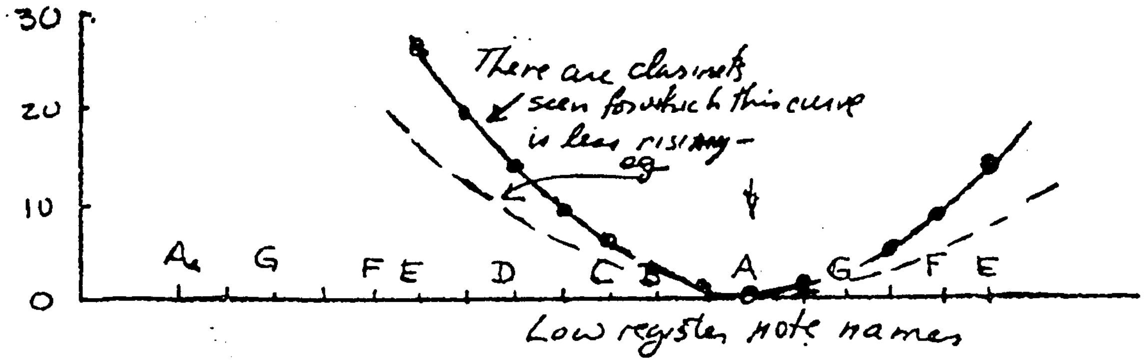
wave velocity is that reduced in a pipe with closed holes
pipe segment "looks" elongated

Wave impedance is that of a smooth pipe with increased cross section
pipe segment looks enlarged

J. SECOND-REGISTER AIR COLUMNS, ETC.

Review FMA, sec. 21.5, p. 455ff, especially the text material on p. 459 about the effect of a misplaced register hole.

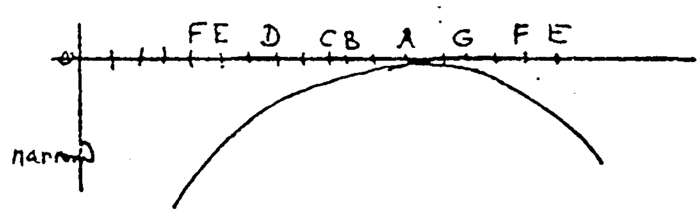
ON A NORMAL CLARINET (back at least to 1800), the register hole is placed close to its ideal position for producing the A3-to-E5 transition into the 2nd register. Mode 2 of the air column is then pulled sharp in the manner sketched for all other notes in the second register:



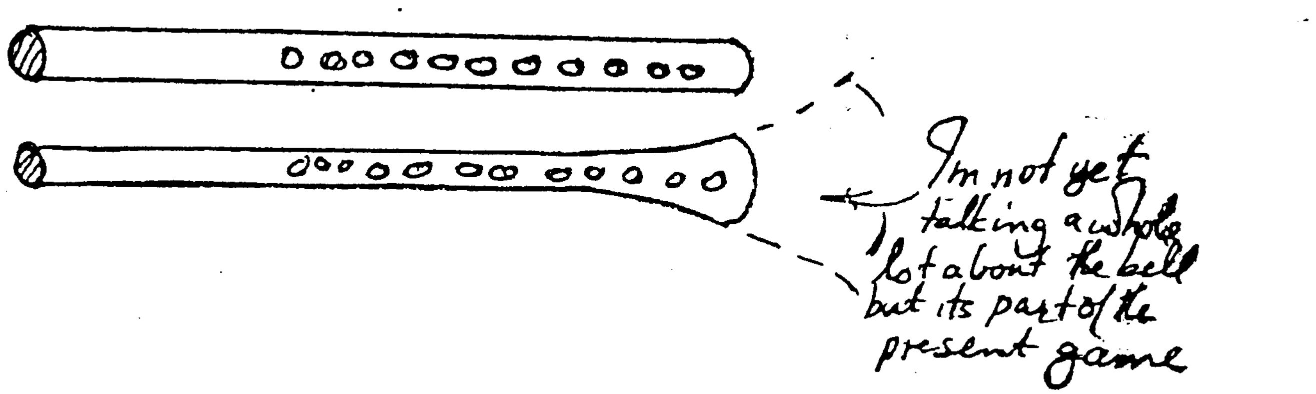
LET US SEE HOW AN HONEST FELLOW WOULD ARRANGE A CLARINET IF HE WATCHED ONLY TUNING OF 12ths AT PIANISSIMO LEVEL SO THAT THERE WOULD BE NO COOPERATION BETWEEN RESONANCE PEAKS. He wants only to get a 3-to-1 frequency ratio between the UNDISTURBED 1st-mode peak and the SHIFTED 2nd-mode peak.

(THIS IS ALMOST EXACTLY WHAT HAPPENED IN THE 1940'S after the invention of God's Gift to the Instrument Maker--the Stroboconn--before people realized that it is a tool and not a master, and that tone and response are important.)

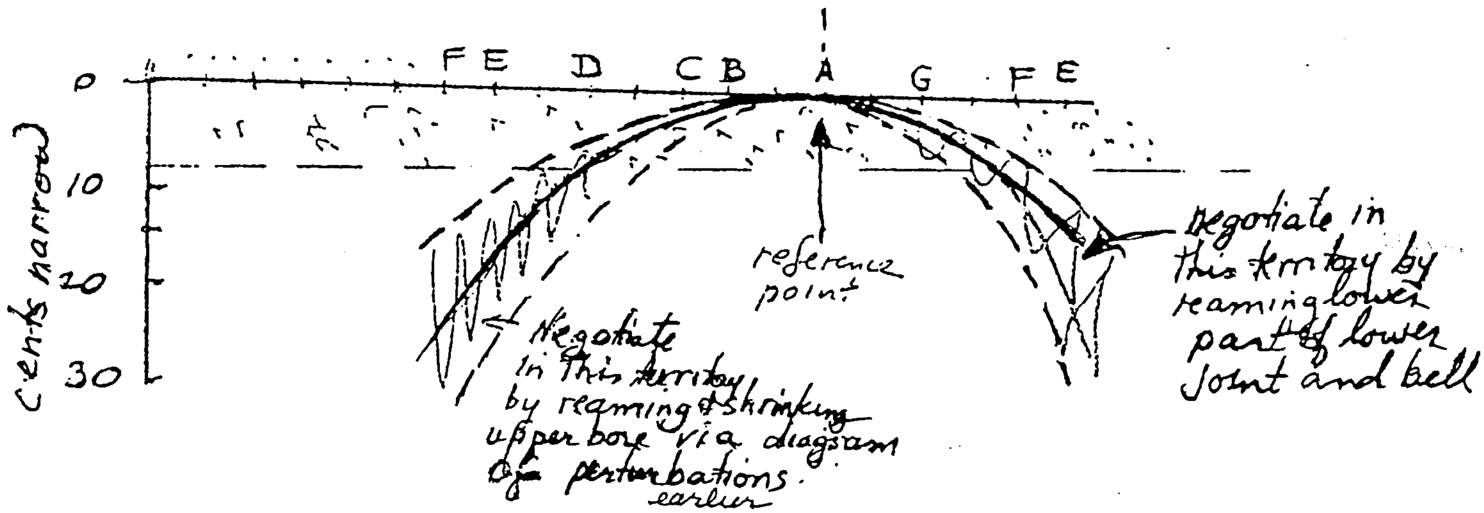
In the spirit of the mode-ratio shrinking diagrams of the section titled To Meet a Clarinet, tuning a clarinet to the specs given above requires that the closed-holes, open-holes, reed-and-mouthpiece, and finally bore-reaming perturbations to the air column itself give rise to a curve that is a mirror image to the one at the top of this page, at least between the E3 and F4 fingerings:



COMPARE THIS WITH THE DIAGRAM IN ITEM 5 several pages back (the AGGREGATE picture with a shaded region). Clearly this earlier curve is qualitatively of the right sort at the north end of the air column, but it is an under-correction, according to our well-intentioned disigner. How do we take care of getting a droop at the south end of the horn? We simply enlarge the bore down at this end:



If we are clever in our reaming, starting somewhere under the right-hand 3rd finger, we can get a droop of the right sort. For various reasons one usually ends up in practice with a drop of around 35-40 cents at the low E frequency ratio.



This represents an overall air column plus tonehole, reed, and mouthpiece and bore profile effects arranged to play a perfect 12th with a fixed embouchure at a pianissimo dynamic level.

In practice, this gives curious tuning of the scale in one register if it is smooth in the other. It also has lousy response in the low register except around D4 to A3. Moreover, it also runs flat on a crescendo in the low register, except around A3.

Now we are in A REAL MESS

At forte levels, the low register runs away from the pitch for one reason. If one tries to get good tone in the clarion register by centering up the reed resonances, it turns out that both ends of the scale run a trifle flat (around 10-15 cents). Some makers put in this bore profile to make the 12ths good at some mf playing level (which makes better practical sense than the scientifically easier ppp-level tuning discussed earlier). But, one still has all sorts of quirks in the tuning, note by note.

<u>A BETTER WAY TO ARRANGE THINGS</u> (fairly common pre 1910 and often in Germany today)

(A) Get the upper bore reamed to put the pianissimo, unlipped 12th about 10-15 cents wide. The exact adjustment is made to put C6 in tune when the register hole is open and the embouchure is slackened to put f_r at the second harmonic (1864 Hz). This will then give a little help to the 6th harmonic of F4 if desired. Tune this pretty much right on at the mf playing level.

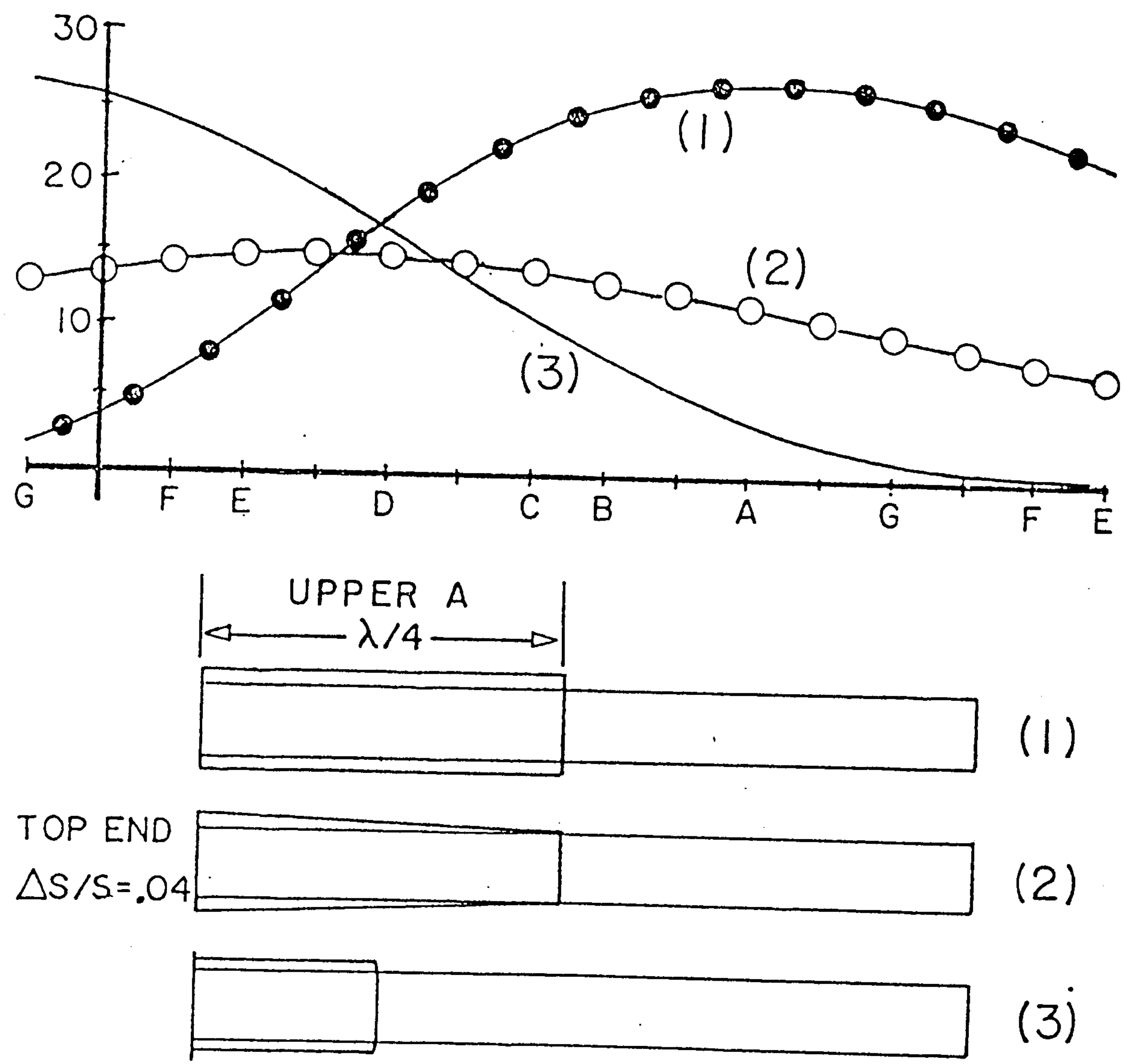
(B) Tinker the lower bore so that only a little strain will let you bring the B4 into tune with the embouchure slackened again, to put f_r at 2200 Hz, where it can feed the 5th harmonic of the tone and so (being an odd harmonic) give it a more clarinetish sound than one might otherwise get from the bell note.

(C) In between these limits, things are straightforward.

(D) NOTE! YOU <u>MUST</u> arrange things so the player can <u>in practice</u> pull up 12-15 cents or down 12-15 cents without loss of security, so as to be able to fit in all chords he meets.

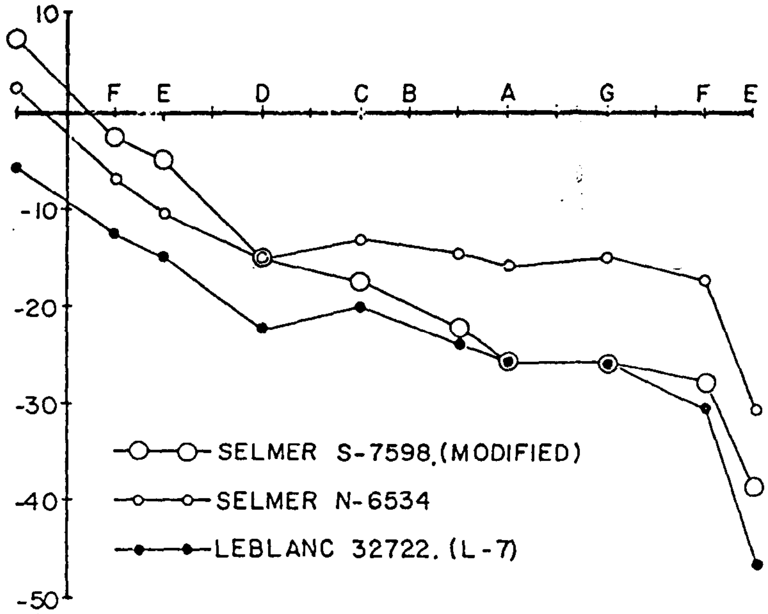
VERY IMPORTANT!!! See FMA, sec. 15.4, p. 258ff, ESPECIALLY THE LAST PARAGRAPH. VERY IMPORTANT!!!

[J4]

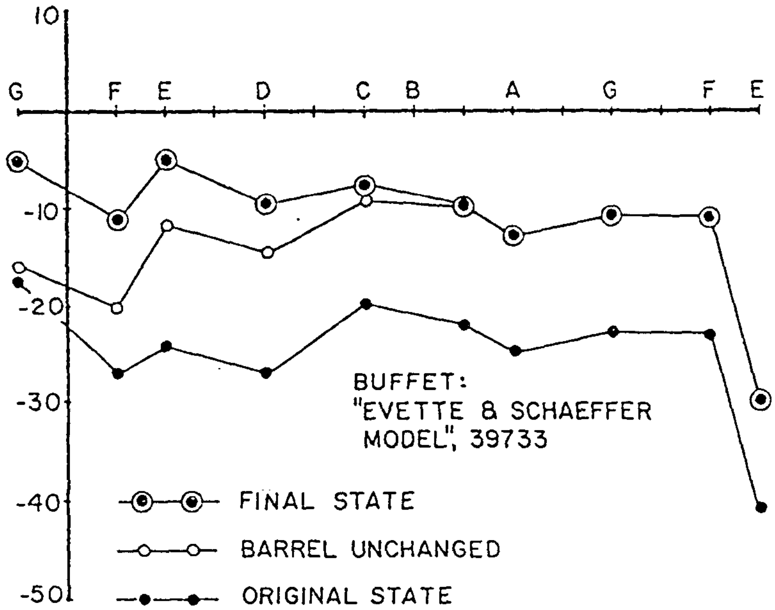


38b

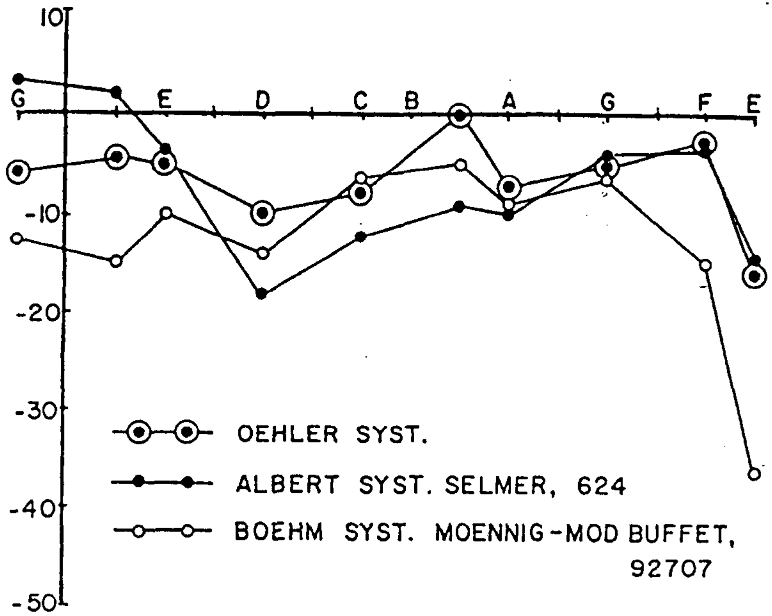
[J5]


10
-10
-20
-30
-40
-50
F E D C B A G F E
SELMER S-7598.(MODIFIED)
SELMER N-6534
LEBLANC 32722. (L-7)

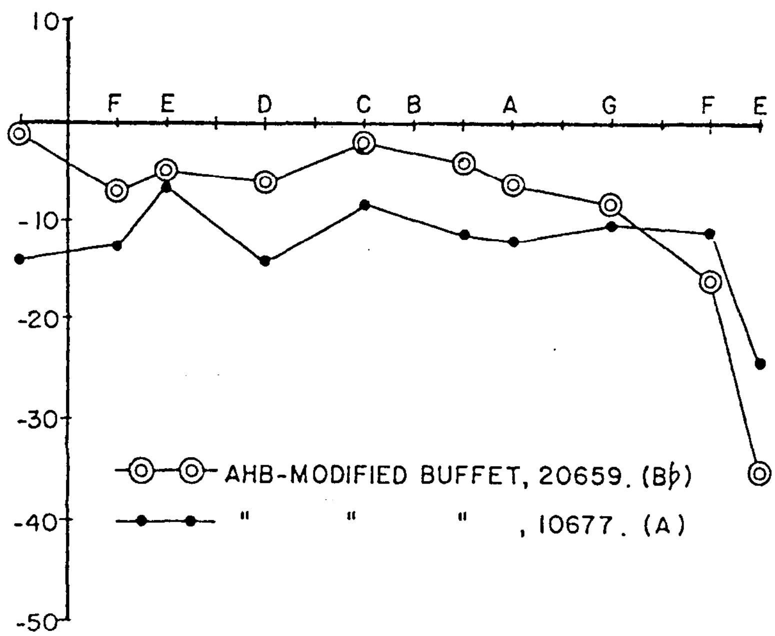
[J6]


10
-10
-20
-30
-40
-50
G F E D C B A G F E
BUFFET:
"EVETTE & SCHAEFFER MODEL", 39733
FINAL STATE
BARREL UNCHANGED
ORIGINAL STATE

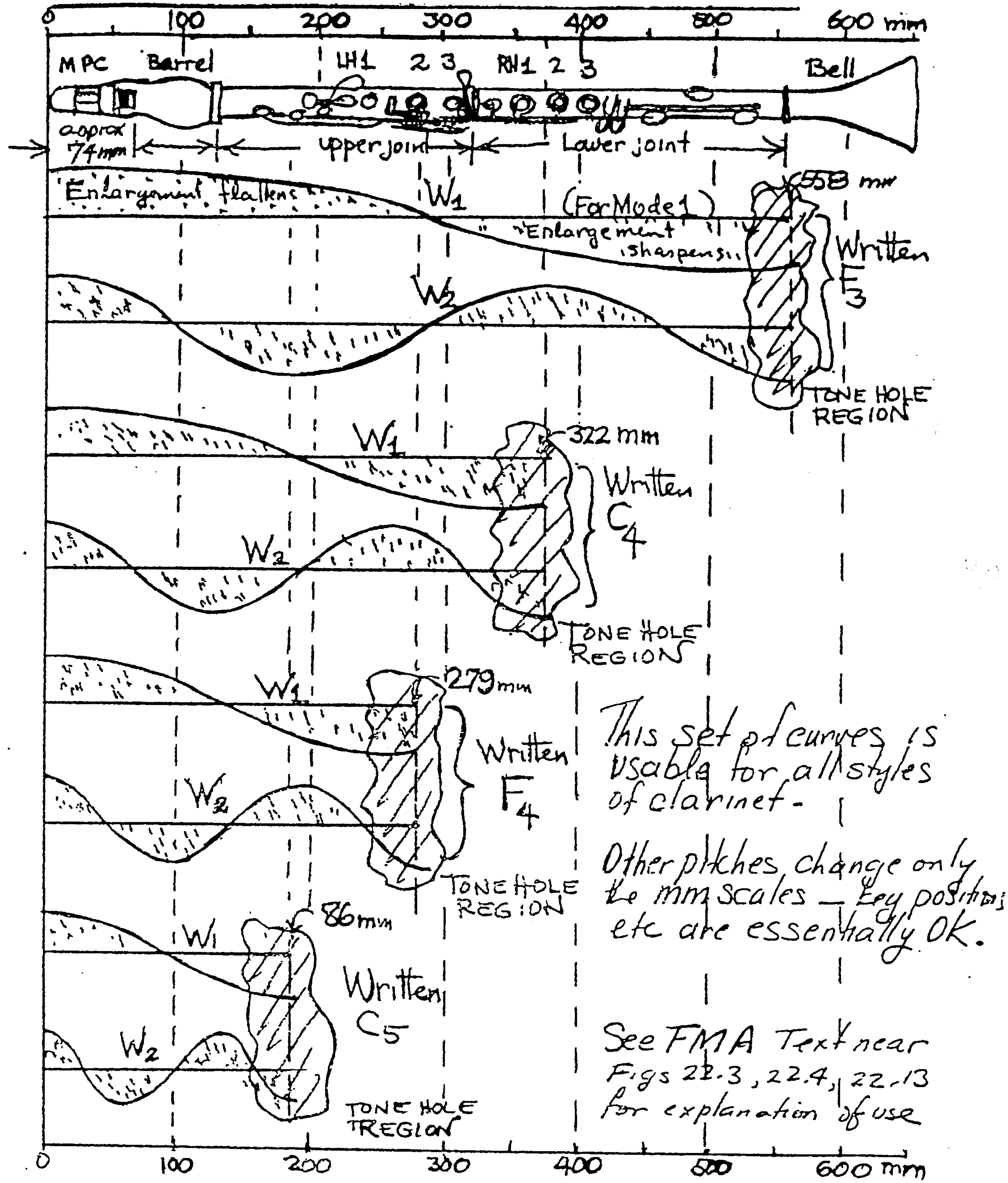
[J7]


10
-10
-20
-30
-40
-50
G
E
D
C
B
A
G
F
E
OEHLER SYST.
ALBERT SYST. SELMER, 624
BOEHM SYST. MOENNIG-MOD BUFFET,
92707

[J8]

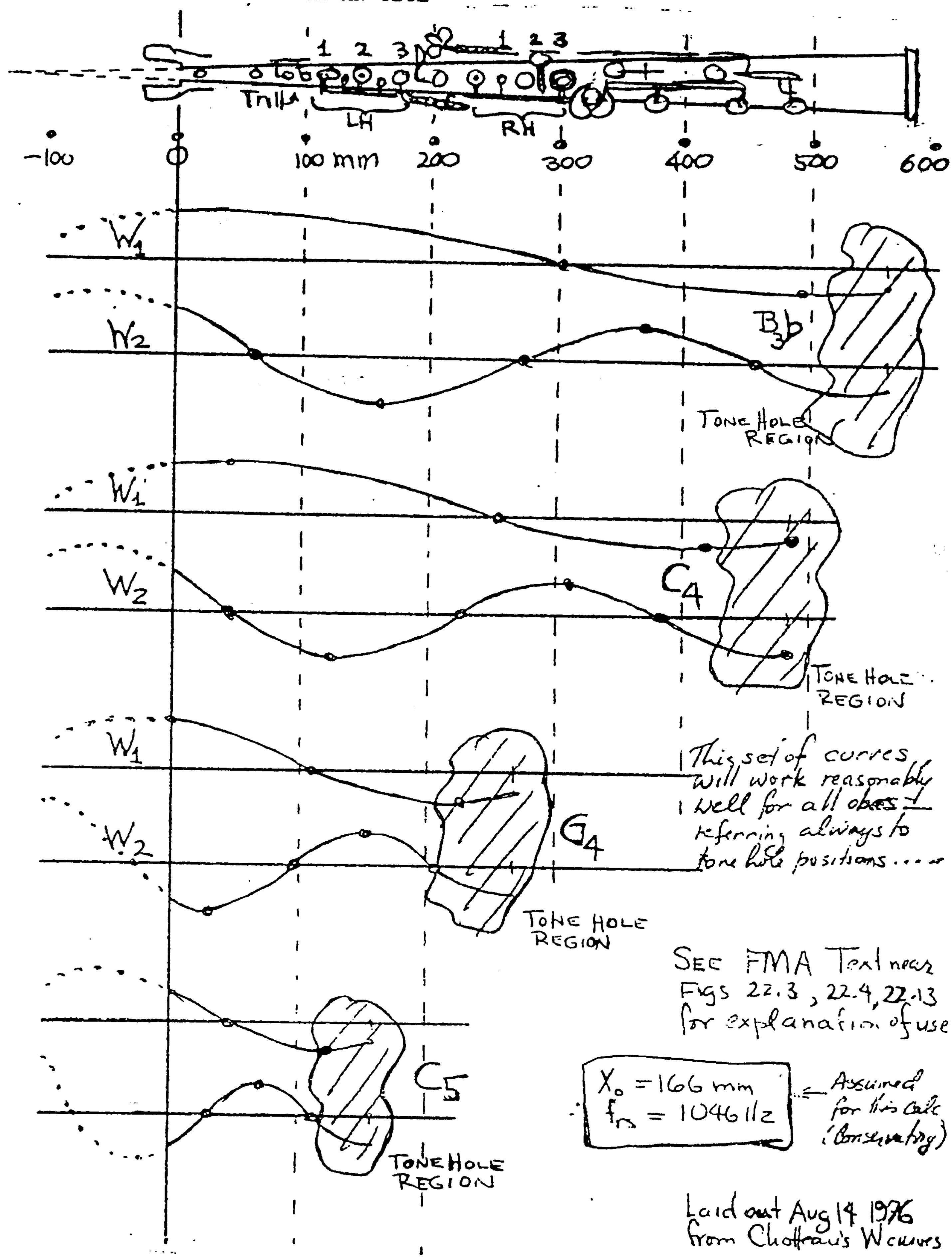

10
-10
-20
-30
-40
-50
F
E
D
C
B
A
G
F
E
AHB-MODIFIED BUFFET, 20659. (B♭)
" " " , 10677. (A)

PERTURBATION W-CURVES FOR A B-FLAT CLARINET



Laid out in calc of Jan 19 1976

PERTURBATION W-CURVES FOR AN OBOE



This set of curves will work reasonably well for all oboes ± referring always to tone hole positions....

SEE FMA Text near Figs 22.3, 22.4, 22.13 for explanation of use

$X_o = 166$ mm
$f_n = 1046$ Hz

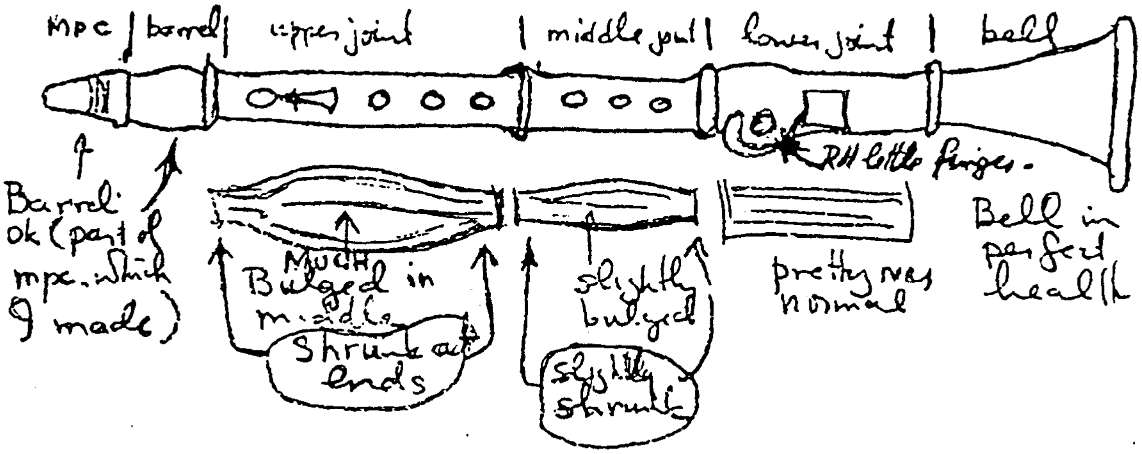
← Assumed for this calc (Conservatory)

Laid out Aug 14 1976 from Chottrau's W curves

ASSIGNMENT 3

(1) Work out question 2 in FMA, sec. 22.8, p. 502. Hints: you will have to make combined use of the information supplied in sec. 22.3 (especially as it bears on Fig. 22.3) and on the sheet following this one.

(2) When I took possession of the boxwood D'Almaine clarinet currently being played by Steve Uhler [a member of this class], its bore was rotted and worn away so that it was enlarged in its mid-regions, section by section, thus:



There were numerous mechanical clues to the original state of this instrument--but the symptoms via players' tests alone were enough to establish conclusively what had gone wrong with the bore. You all know that the instrument is today in extremely good health, with the bore lacquered in and re-reamed to match both tradition and good response and tuning.

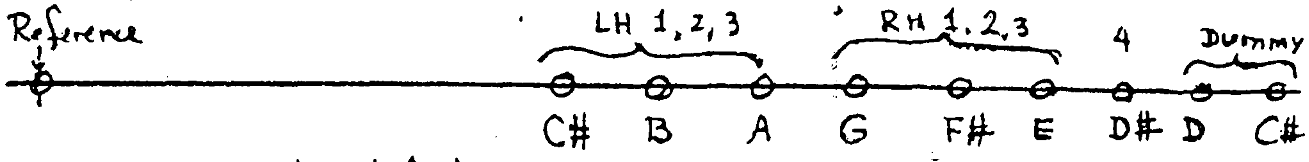
Figure out what tuning and tonal clarity, centered or easy response, etc., etc., symptoms you would expect to find on the lower and second registers of this clarinet in its pre-repair state. Note: the tone holes were in perfect health.

IMPORTANT! The bulges and shrinkings indicated above are not referred to a straight-sided cylinder, but rather to departures from the properly adjusted original bore profile.

Use the information referred to in question 1. Also you may need to refer to Baines to get a proper orientation regarding the R-H finger holes--which holes on a Boehm clarinet correspond to these? How about the R-H little finger hole?

K.

COMPUTATIONS FOR FLUTE LAYOUT (pseudo-baroque), <u>REVISED</u> VERSION OF LAYOUT IN D, SEC. (A), p.16.



New hole diam = 8mm 6.4 6.4 4.75 7 5.6 6.4 6.4 6.4

$2a = 17$mm wall thickness 2.2mm $t_e \cong t + .75 \cdot (2b)$

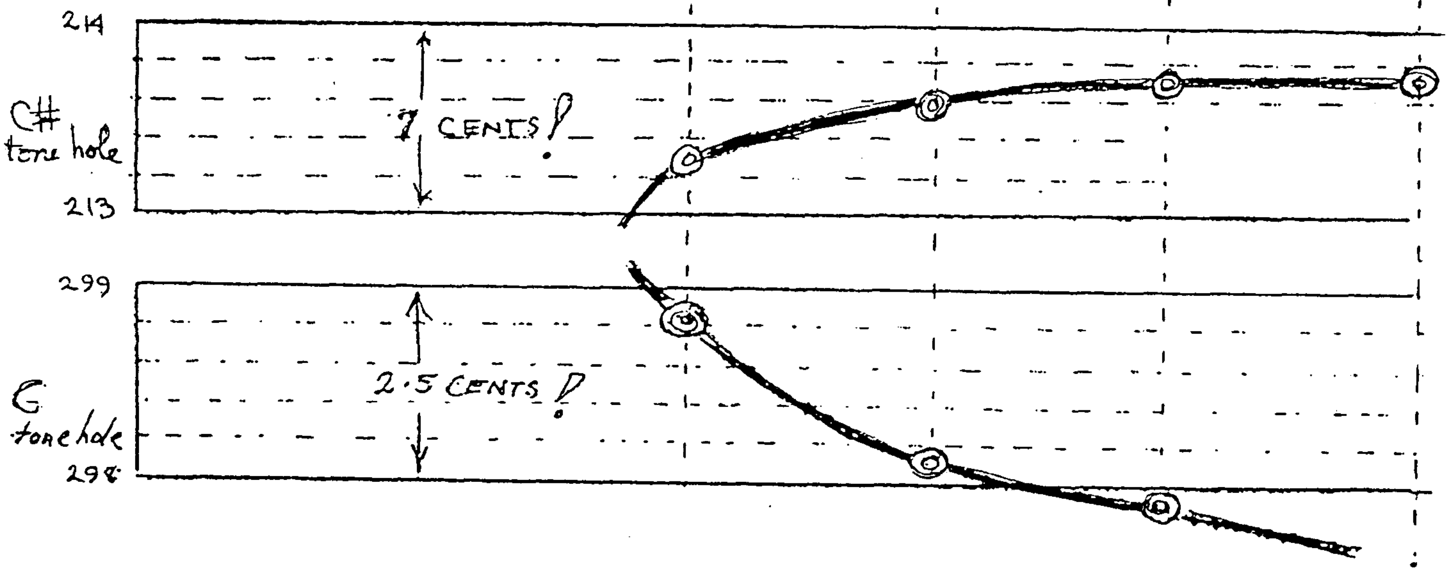
$$C = \left(\frac{2s}{2}\right)\left[\sqrt{1 + [4t_e(a/b)^2](1/2s)} - 1\right]$$

$4t_e(a/b)^2 =$ 295.1 for 4.75mm hole; 235.9 for 5.6mm; 197.6 for 6.4mm; 175.8 for 7mm; 14.8 1 for 8mm holes. [STO 1 .... 5]

Enter 2S in register 0. RCL $4t_e(a/b)^2$ from 1 to 5, then run as follows: RCL 0, ÷, 1, +, √, 1, −, RCL 0, X, 2, ÷, to calc C. using HP.25

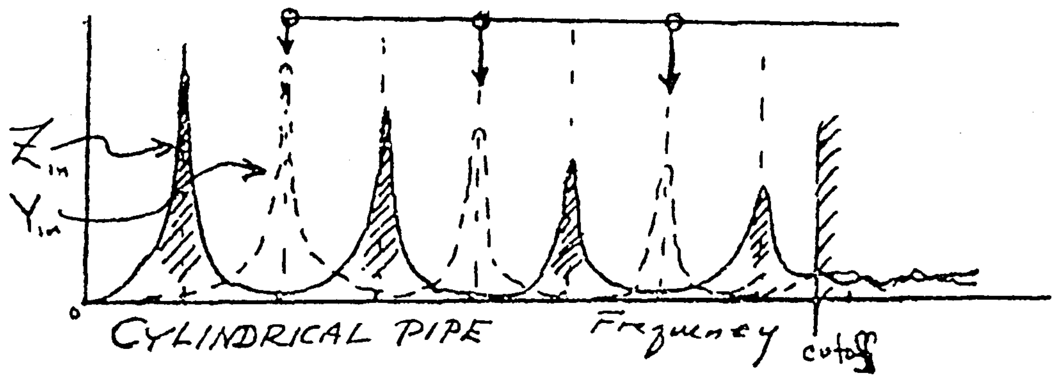
	Normal length	initial 2s	hole size	C_a	initial hole pos	new 2s	C_b	hole pos	2s	C_c	hole pos	2s	C_d	hole pos	2s	C_e	hole pos
C#	233.1	28.6	8	21.2	211.9	22.9	19.8	213.3	21.5	19.5	213.6	21.5	19.4	213.7	21.2	19.4	213.7
B	261.7	32.0	6.4	26.9	234.8	31.1	26.6	235.1	31.2	26.6	235.1	31.6	26.8	234.9	32.1	26.9	234.8
A	293.7	36.0	6.4	27.8	265.9	34.3	27.4	266.3	32.5	27.0	266.7	31.4	26.7	267.0	30.9	26.6	267.1
G	329.7	19.6	4.75	29.5	300.2	22.2	30.9	298.8	23.6	31.6	298.1	24.0	31.8	297.9	24.8	32.1	297.6
F#	349.3	42.3	7.0	26.9	322.4	42.4	26.9	322.4	41.4	26.7	322.6	40.8	26.6	322.7	40.5	26.5	322.8
E	392.0	23.3	5.5	27.2	364.8	25.8	28.2	363.8	27.0	28.6	363.4	27.4	28.8	363.2	27.6	28.8	263.2
D#	415.3	24.7	6.4	24.7	390.6	24.2	24.5	390.8	24.1	24.3	390.8	24.1	24.5	390.8	24.1	24.5	390.8
D	440.0	26.2	6.4	25.2	414.8	25.8	29.1	414.9	25.8	25.1	414.9	25.8	25.1	414.9	25.5	25.1	414.9
C#	466.2	27.7	6.4	25.6	440.6	27.3*	25.5	440.7	27.3*	25.5	440.7	27.3*	25.5	440.7	27.3*	25.5	440.7

<u>EXAMPLES OF THE "SETTLING" OF HOLE POSITIONS</u>



L. INTRODUCTION TO THE FLUTE

Recall discussion of the flute in sec. C and in FMA, sec. 22.6, p. 489. The flute runs via cooperations between Z_{dips}, i.e., via cooperations between Y_{peaks} wher $Y = 1/Z$ is the admittance (see discussion of Fig. 22.14, FMA, p. 498).

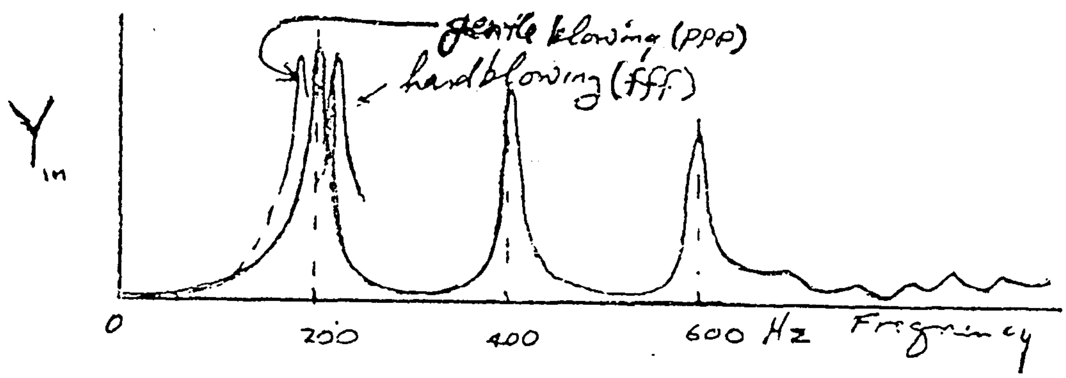


In a reed woodwind we find that the instrument plays a little lower in frequency than what we would naively expect from a calculation of the effect of reed elasticity alone on the natural frequency of each air column resonance. To take care of this we found it worthwhile to define a reed cavity equivalent volume (FMA, Fig. 22.1, etc.) that is big enough to represent the extra lowering of the playing frequency. This works because V_{eq} is approximately independent of frequency and so can be coped with just about like a hard-walled cavity.

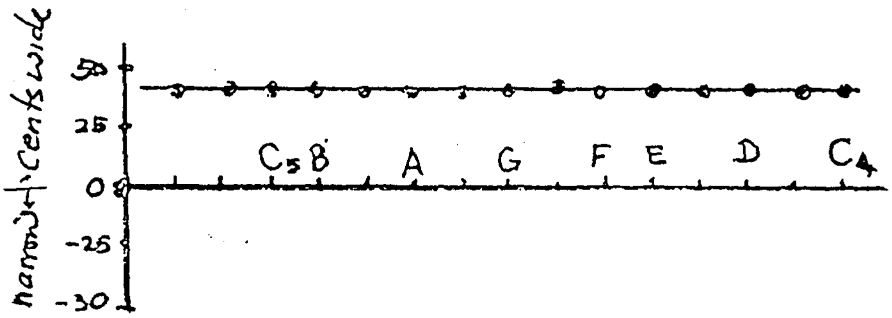
In the flute tribe we find that the instrument plays above or below the corresponding admittance-peak frequency depending on the strength of blowing (see lower half of R-H column, FMA, p. 495).

A FUNNY THING

The resonance peak (of admittance) associated with the playing frequency of a flute note is moved around because of the presence of the air jet at the embouchure hole--BUT, NOT THE OTHER PEAKS (well, almost not hardly a little bit).



Here is the gut issue of making a good flute: Suppose we have a pipe whose lab-measured admittance peaks are at 200, 400, 600 Hz. Peak 1 can move from ≈ 194 to 204 Hz (about a semitone) from ppp to fff blowing. Only at mp blowing are the other peaks lined up to add cooperation to the regime! However, <u>at this level cooperative effects are hardly noticeable</u>! The net effect of the phenomena just outlined is that a good flute that is to give a gutsy, well-centered sound at an mf-to-ff dynamic level (in the low register) must be built so that peak 2 must always be 30-40 cents wider than 2 to 1 relative to the frequency of peak 1 for each note. Tune peak 2 to (the desired playing frequency x 2) and then blow hard enough to pull peak 1 up to the desired playing frequency. If we plot curves of the (f_2/f_1) frequency ratio over the low-register scale, as done in the clarinet sheets preceding these notes, what we would like is



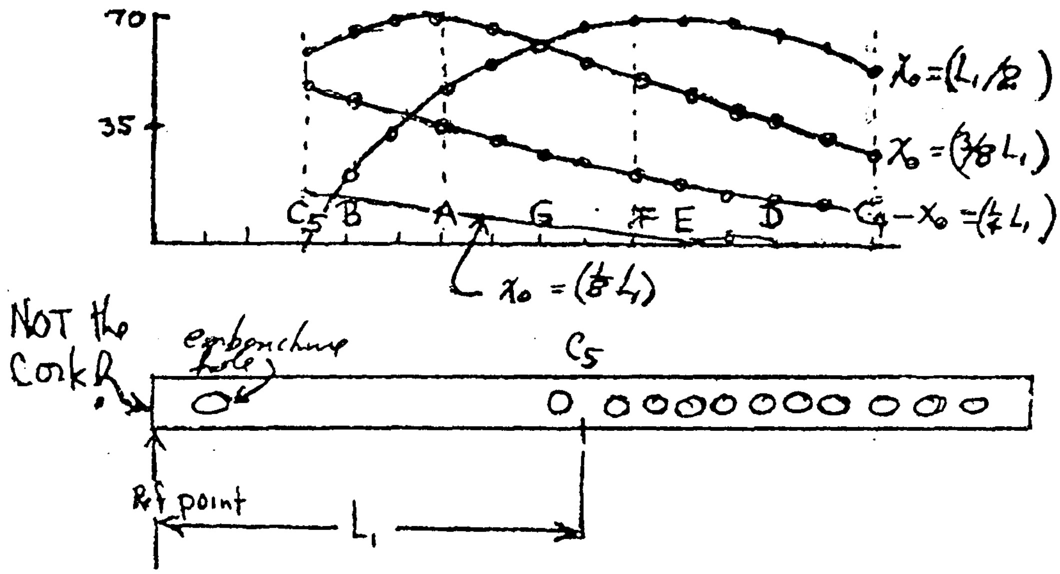
[There is a marginal indication here that the diagram just above applies to ALL flutes, cylindrical or conical, whereas the next one is for cylindrical flutes only.]

In the clarinet air colum notes in section H, I gave the effects of reaming at the mouthpiece end of the bore. Here are some analogous curves (etc.) for a flute tube (a doubly open pipe):

$$\frac{\Delta f}{f} = +\int \frac{S_p}{S_0} \cos \frac{2n\pi x}{L}\, dx$$

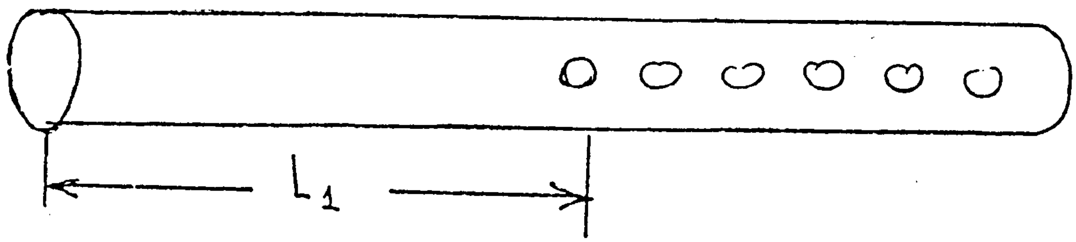
↑ note sign!   ↑ note the 2n in place of the (2n + 1)s belonging to the clarinet tube

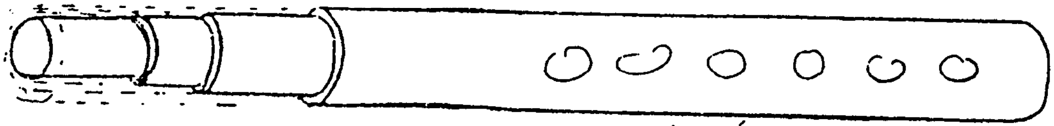
See W-curves for the flute, FMA, Fig. 22.13, p. 496.



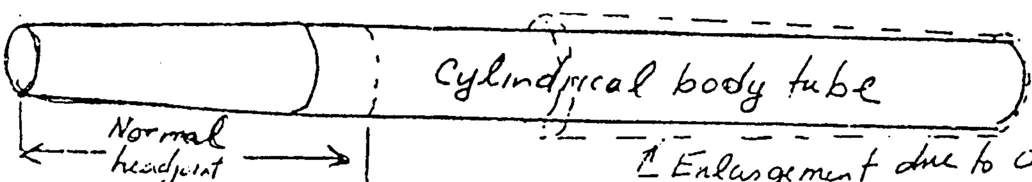
The curves were calculated (HP-25 program on request) for a 10%-diameter reduction at the upper end of the bore a certain fraction of the length L down to the normal position of the C_5 tone hole.

Clearly, we require a sum of contractions to make the desired mode-ratio stretching:

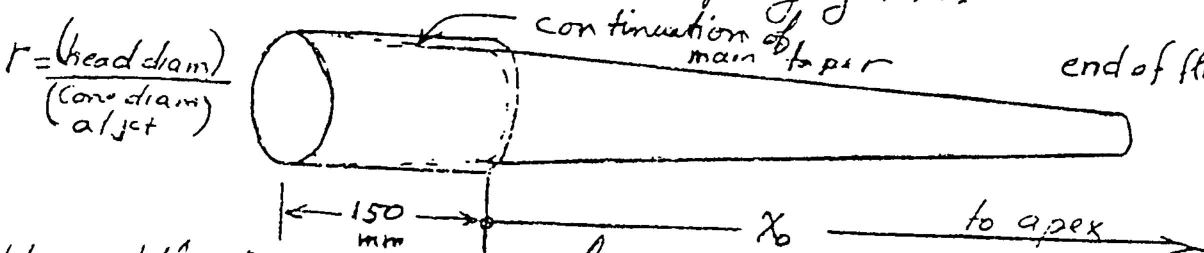




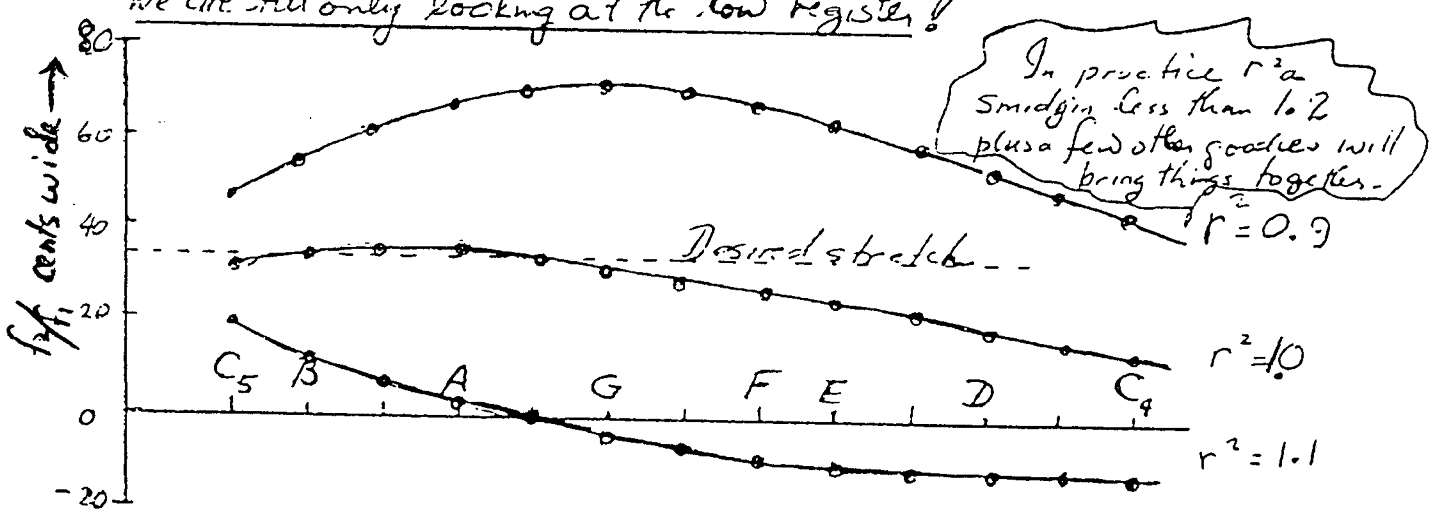
This is the basic idea of the headjoint taper on a Boehm cylinder-bore flute. But......!



THERE IS ANOTHER TYPE OF FLUTE, with basically CONICAL body (see FMA Fig 22.11, p.493 and accompanying text.



We are still only looking at the low register!



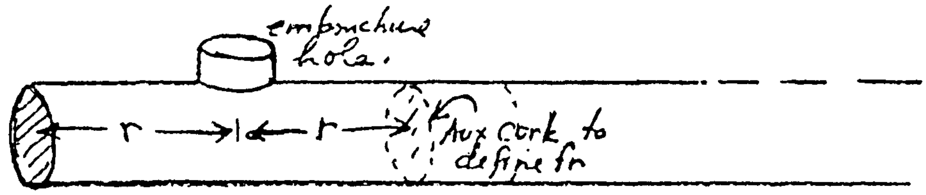
Cylindrical body flutes

Conical flutes with cylindrical heads

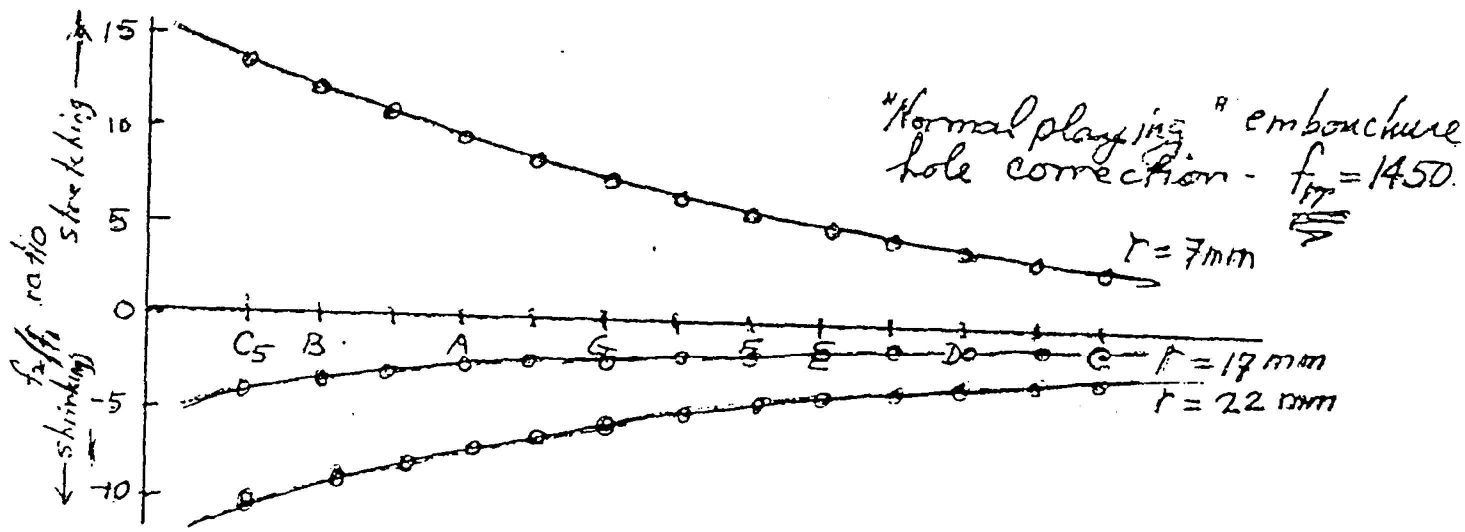
M. ON THE FLUTE (PART II)

We have just seen two ways in which one can get the needful mode-stretching for a flute by means of a contraction of the main bore in the region of the head. Let us now look at some of the other ingredients that fall into the pot when we try to cook up a flute.

Consider what happens at the embouchure-hole-and-cork end [see FMA, p. 495; also see Benade and French, JASA 37 679)1965), Eq. 8, Fig. 3.]

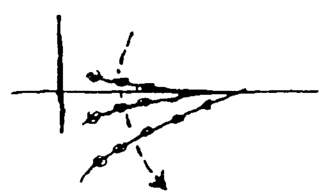


Here f_r is the measured natural frequency of a "bottle" made of bore length 2r, opened in the center via the embouchure hole, with lips in place and using a special blowing technique. Normal f_r is a little above 1450 Hz; f_r = 1600 Hz when a very much rolled open style of playing is used, and f_r = 1250 Hz if a very covered style is used. These are all defined for r = 17 mm, which is usual for the Boehm flute. We will use these to <u>define</u> the lip position, regardless of where we actually put the cork--i.e., we use f_{17} as the reference.



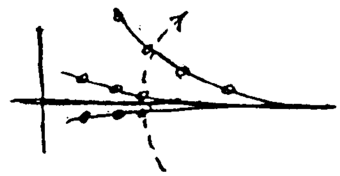
Note: since the stretch or shrink ≈ (freq)^2 approximately, the f_4/f_2 stretch or shrink that matters in the second register is four times as big as what is tabulated here.

For a very open embouchure style, f_{17} = 1600 Hz, the curves look similar but are turned "one notch" ~~clockwise~~ counterclockwise

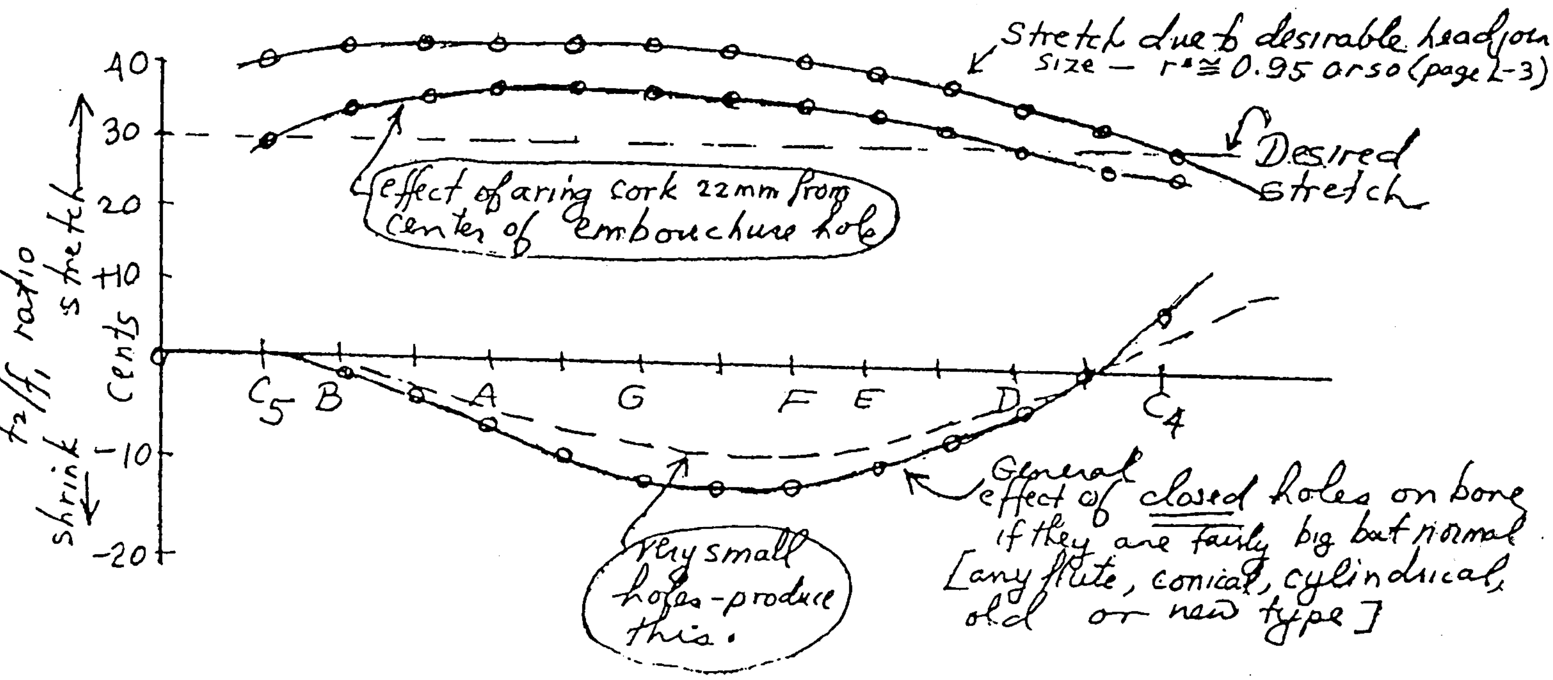


[Note by AHB: [illegible] "drawn backwards." [illegible]]

And for a very closed embouchure style, f = 1250 Hz, the curves are turned ~~counter~~clockwise:

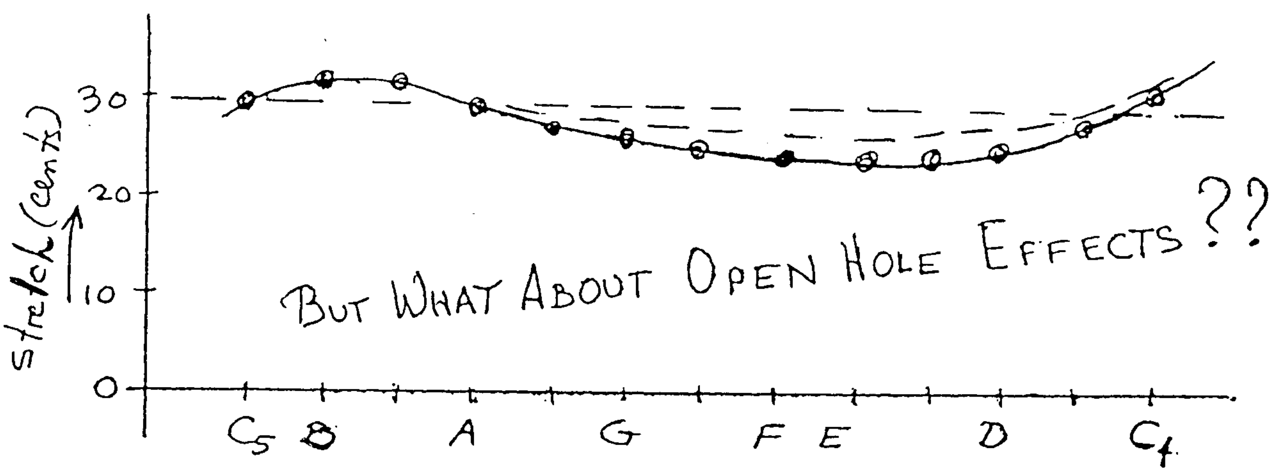


Let us put this together with the curve at the end of section L:

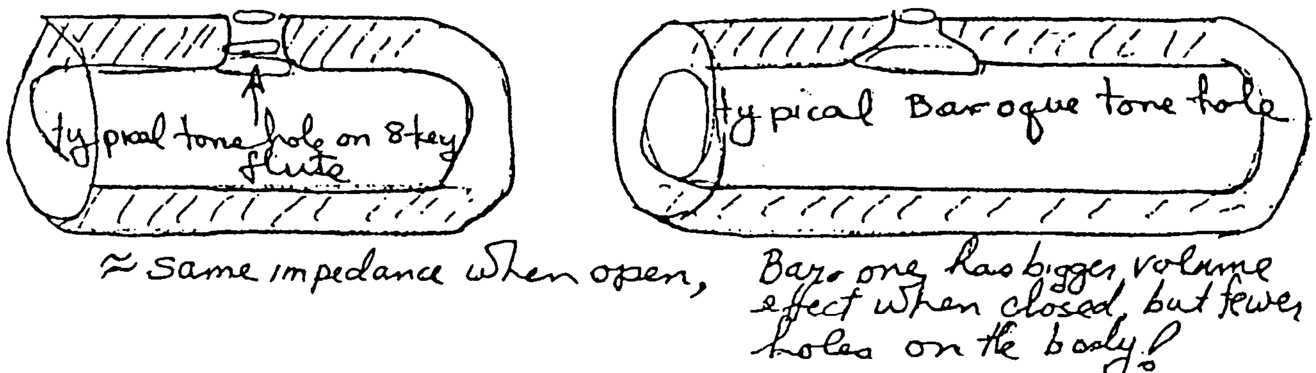


Notice that the toneholes (closed, that is) tend to leave the flute undercompensated in the region between D4 and G4 and slightly overcompensated in the region of C4. Depending on the flute, this latter overcompensation (i.e., an overstretching of f_2/f_1) begins near D#. On some flutes it doesn't begin till C4 itself.

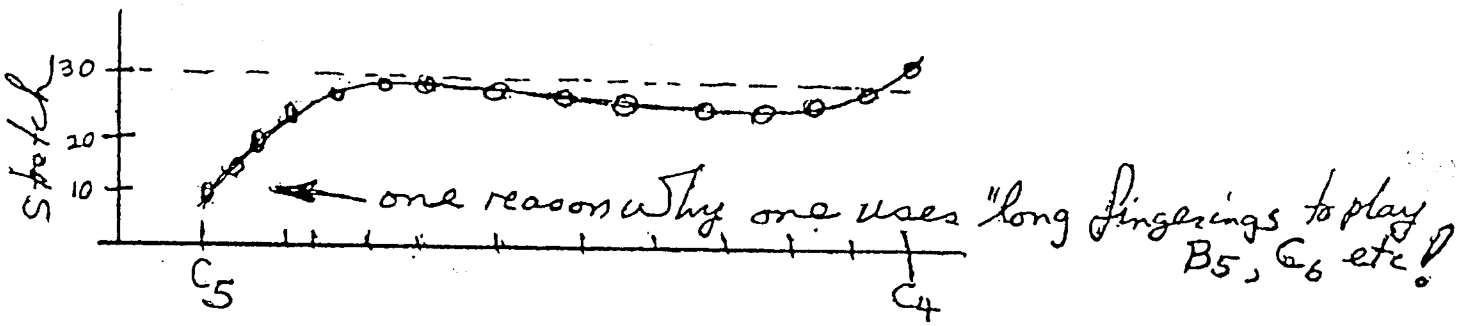
<u>Summarizing everything so far:</u> cylindrical headjoint + long cork position on a conical flute does a pretty good job with a net effect as follows:



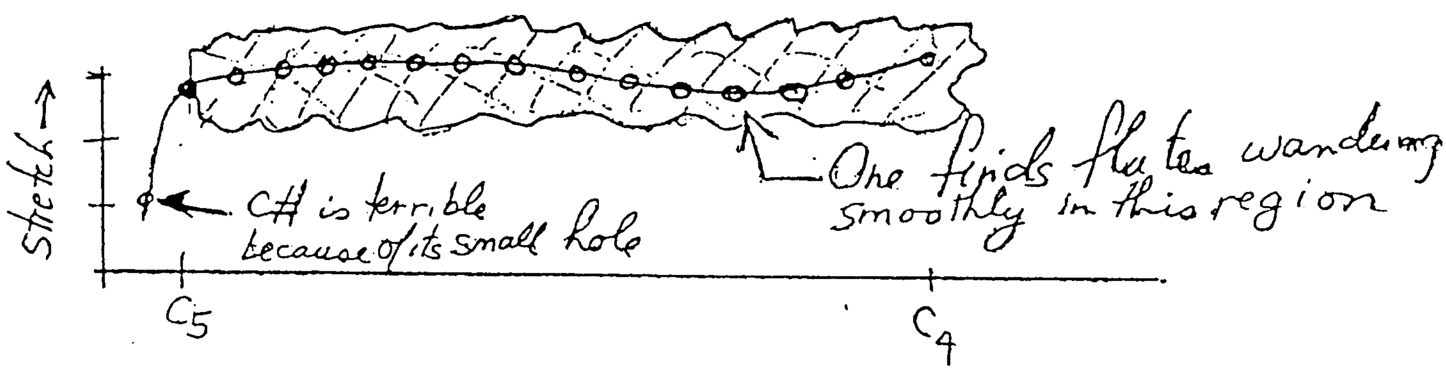
For the moment let us confine our attention to the baroque flute and/or its 6- or 8-key descendants (whose mildly undercut toneholes have roughly the same acoustic relation to the bore as do the small but strongly undercut holes of the baroque flute):



Just as we saw at the upper end of the clarinet low-register scale, there is an f_2/f_1 shrinking due to open tone holes. For a baroque flute, then, the overall f_2/f_1 ratio curve comes out something like this:



On the Boehm flutes (cylindrical and conical) we get something like this:



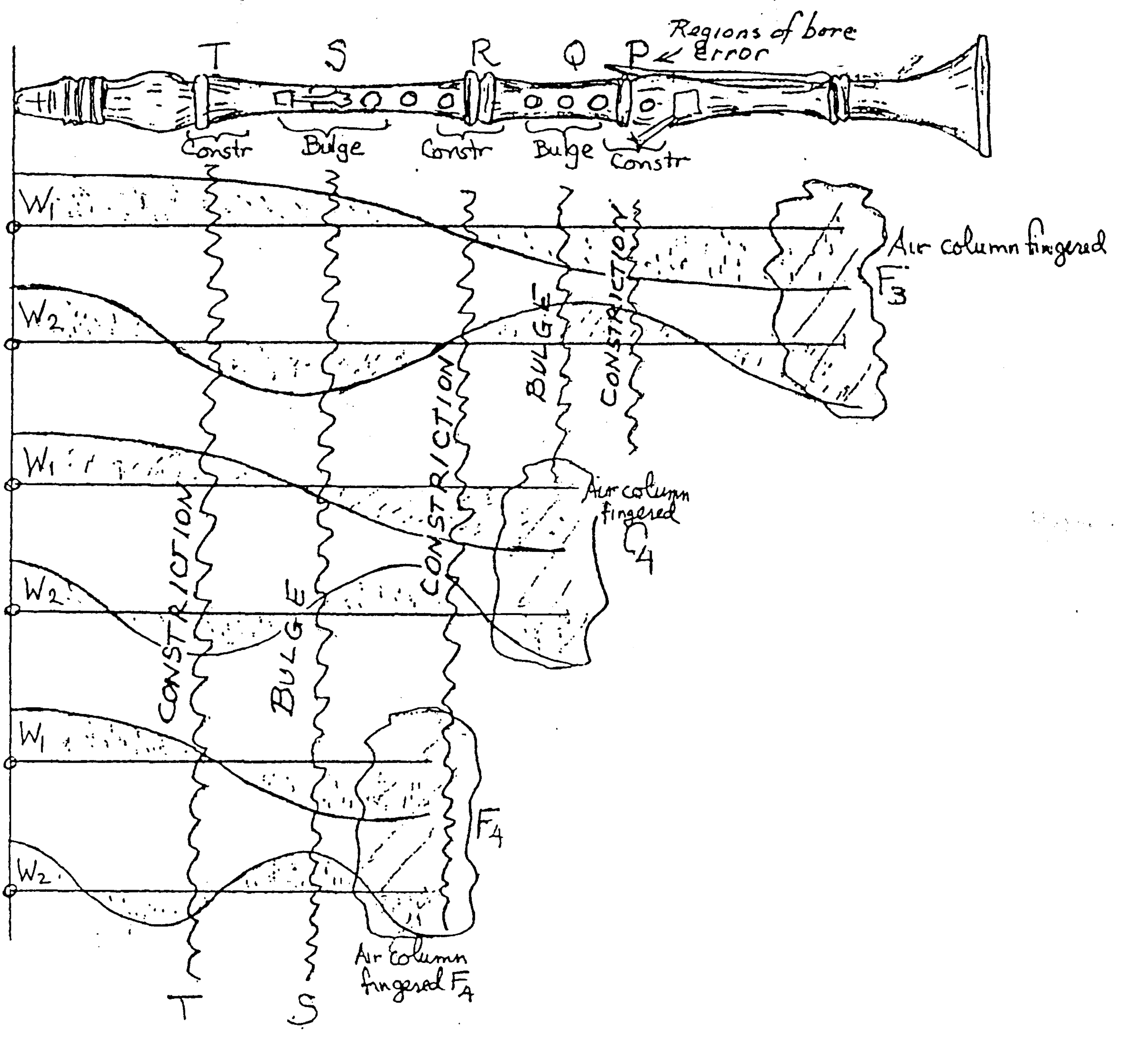
Finally, "big-voiced" flutes have a pretty uniform f_2/f_1 strech of as much as 40 cents.

And "small-voiced" flutes have a pretty uniform f_2/f_1 strech of as little as 25 cents.

Both limits can work very beautifully in their own territory. Big-voiced is more useful soft than small-voiced when it is pushed to loud. (I will demonstrate.)

N.

DIAGNOSIS OF D'ALMAINE CLARINET (see Assignment 3)



Details of other fingerings not shown on the above diagram:

<u>Not worked out here, but: For air column fingered A4</u>

J raises 1 and 2
T lowers 11 some, raises 2
Overall, quite sharp A4, <u>very</u> clear and beautiful tone
f_2/f_3 = 3, drifts slightly shrp on crescendo--pretty obvious on initial test

<u>For air column fingered F3</u>

P lowers 1, raises 2
Q raises 1, lowers 2
R does nothing much
S lowers 1, raises 2
T raises 1, lowers 2
Overall, little change (good note, drifts <u>sharp</u> on crescendo)

<u>For air column fingered C4</u>

Q slightly raises 1 and 2
R lowers 1, raises 2
S has little effect
T raises 1, lowers 2--DOMINANT

Overall, little change, drifts flat on crescendo; G5 slightly flat

<u>For air column fingered F4</u>

R lowers 1 and 2
S raises 1, lowers 2
T makes little change for 1, lowers 2
Overall, sharp F4, unclear tone, very flat C6

<u>Not worked out here, but: For air column fingered A3</u>

P lowers 1 and 2
Q raises 1, lowers 2
R raises 1, lowers 2
S has little effect
T raises 1, lowers 2
Overall effect is spectacular! A3 is a very sharp, dead tone; E5 is very flat. This particular diagnosis was spectacular in the unrepaired state of the instrument.

O. ASSIGNMENT 4

(1) The valve set of an otherwise perfect trumpet sometimes "looks undersized" to the rest of the air column. Use the perturbation curves on the next page to figure out why this undersizing makes the following symptoms for the "open" (no valve buttons pressed) horn.

(a) C4 plays sharp at a ppp level and ~~runs flatter~~ stays ≈ the same (on correct perturbation curves). on crescendo ~~but never makes it down to true pitch~~. (Hint: consult the effects on resonance peaks 2, 4, 6, and 8. [Changes made in AHB's teaching set]

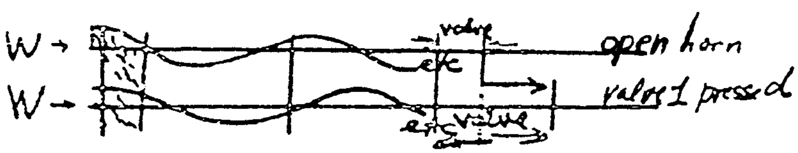
(b) G4 plays flat at a ppp level and drifts sharp on crescendo. (Consider peaks 3, 6, 9 in your analysis.)

(2) I have a baritone horn (currently being modified) which has a bore error that translates to an undersized region in the 60-mm area just south of the valve set (of a trumpet). Figure out the playing consequences of this error if the otherwise perfect instrument sounds C4, G4, and C5.

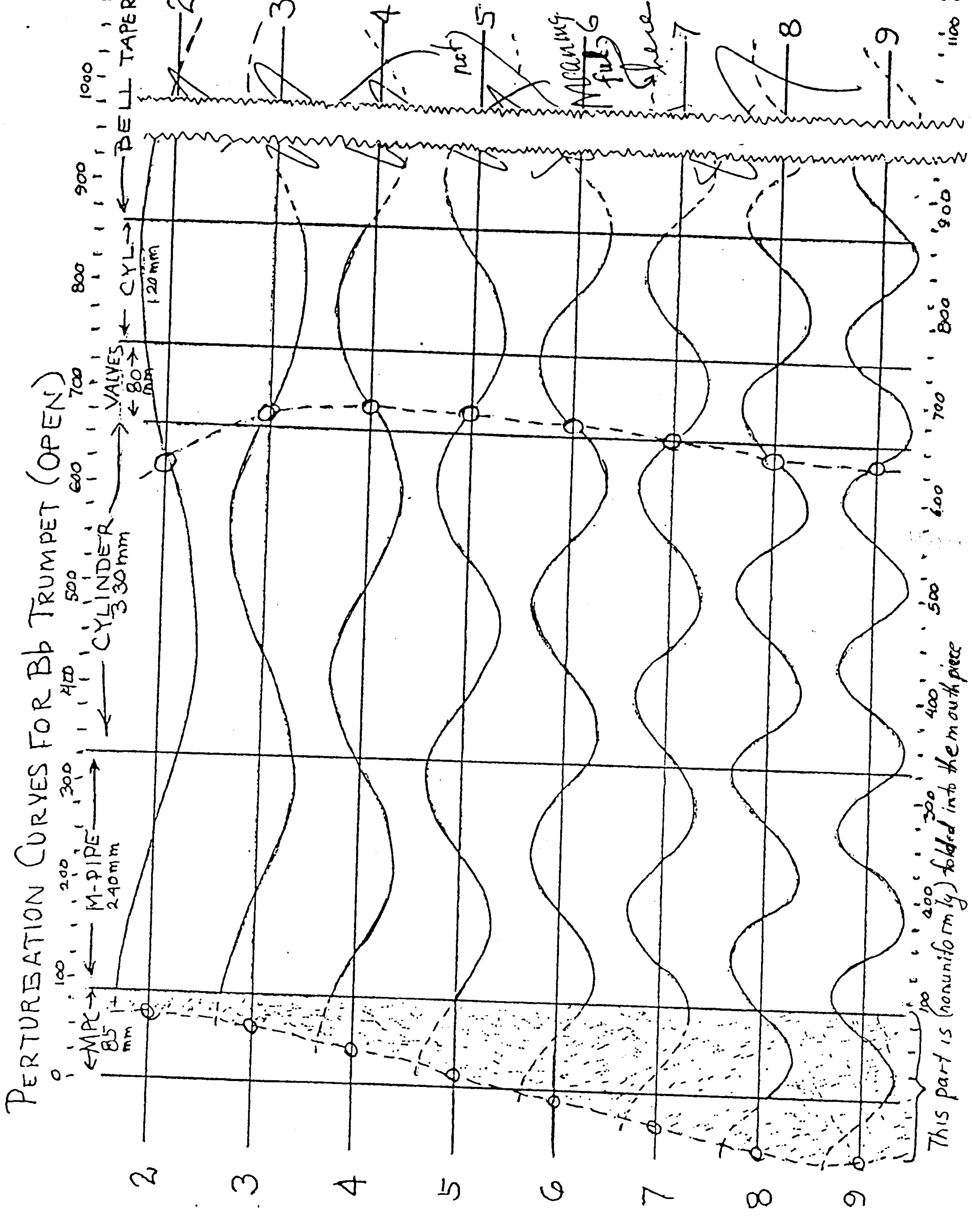
(3) Another error on this baritone corresponds to an oversize region in the 60 mm just north of the trumpet's valve set. Taken by itself, what playing consequences does this have for C4, G4, and C5? [Answer:] All have lousy response

(4) What is the overall effect of the two errors (taken together) that are dealt with in problems 2 and 3?

(5) Perturbation curves can also be calculated for an air column in which one or another of the valves is pressed. On a trumpet, pressing valve button 1 adds about 181 mm to the (approximately) 80-mm length of the valve set of an open horn. Draw a careful freehand sketch showing on one line the W curve for the open horn taken from the dittoed sheet [does this refer to the perturbation curves on the next page??--VB], and on a line directly below, the W curve you would expect for the instrument with the first valve pressed.

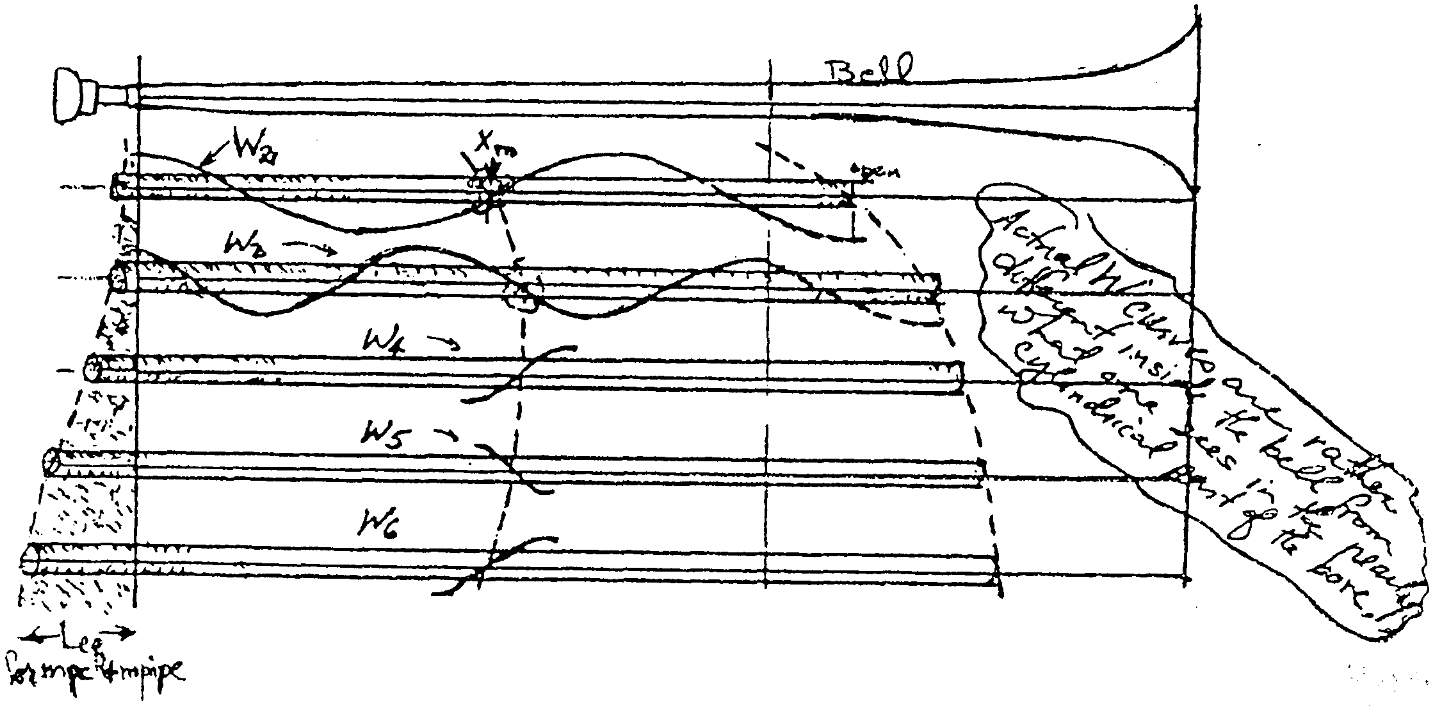


See FMA discussion around Fig. 20.6, p. 403. Look at Fig. 20.8 but recall these are not W-curves. Read the numbered statements in sec. 20.7 and the text immediately following. All the questions depend heavily on our earlier use of W-curves on oboes, flutes, and clarinets. The D'Almaine clarinet example is most closely related.


PERTURBATION CURVES FOR Bb TRUMPET (OPEN)
MPC 85 mm
M-PIPE 240 mm
CYLINDER 330 mm
VALVES 80 mm
CYL 120 mm
BELL TAPER
This part is (nonuniformly) folded into the mouthpiece
not meaningful here
2
3
4
5
6
7
8
9

P. MISCELLANEOUS BRASS-INSTRUMENT INFLUENCES--PART I

Consider a trumpetlike air column:



The cylindrical pipes in this figure stand for the "equivalent" cylinders that have the same natural frequencies as the real trumpet. The left end is located at the point near the mouthpiece where the mouthpiece plus mouthpipe L_{eq} reaches. The right end is located in the manner indicated in FMA, Fig. 20.8, p. 407, for an ideal Bessel horn (with simple closure at the small end).

Notice that at a point in the real trumpet marked X_m is a sort of acoustical midpoint--the perturbation curves all behave like $\sin\ 2k_r(x - x_m)$ in the immediate neighborhood of x_m.

(a) A symmetric enlargement or contraction centered at x_m has zero effect on the natural frequency.

(b) An enlargement or contraction just to one side of x_m has opposite effect on the even-numbered mode frequencies as compared with the effect on the odd-numbered modes.

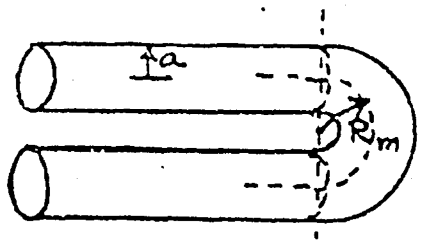
(c) On a real brass instrument, the x_m for all the modes are pretty much at the same spot.

SEE THE PERTURBATION CURVES FOR THE B-FLAT TRUMPET (p. 55 of these typed-up notes). On the figure, the circles and dotted curve trailing down across the sheet, just to the left of the valve set, indicate x_m for the typical trumpet.

Side remark: If there is a systematic enlargement or contraction in the valve set, centering it at an average value of x_m means that the perturbation will have no influence on the mode relationships, regardless of the number of valves pressed! This may or may not be useful.

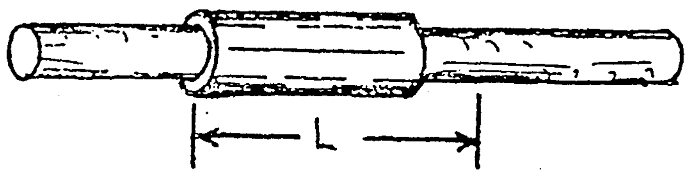
The question often arises as to the effect of sharp bends in the bore.

The valve crooks of most brasses and the bow at the butt end of a bassoon are proportioned to make $(R_m/a) \approx 3$. Note that the sax ratio is slightly more.



The numerical value of the effect is not totally clear at present, but we know that in the same way that closed toneholes enlarge and elongate the bore: the bend looks to the trumpet like an enlargement and a shortening.

A bend of midline length L and radius a "looks" thus:

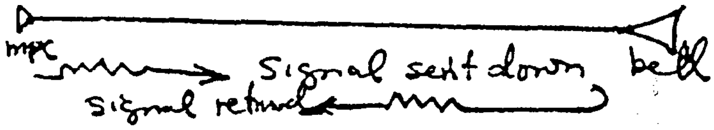


The percentage change in equivalent length and equivalent cross-sectional area $(\pi a^2)_{eff}$ is the same--typically 5-10 percent, it would seem. (Doug Keefe and I are getting set to remeasure this in clear-cut surroundings.) See FMA Digression on p. 409, plus attached reference.

So far we have dealt only with the effect of cross-section changes on the natural frequencies of the horn, and thence on its steady-state playing behavior.

LOOK AT THE START-UP BEHAVIOR AS IT IS AFFECTED BY BORE DISCONTINUITIES. (See FMA, sec. 20.9, p. 425ff.)

The player starts his tone by launching a signal down the bore, hoping that what comes back will arrive properly in step to help the regenerative system build up the tone.



What if there is a discontinuity somewhere? [Other things to consider:] Early reflection--perhaps modified shape also for the returned signal.

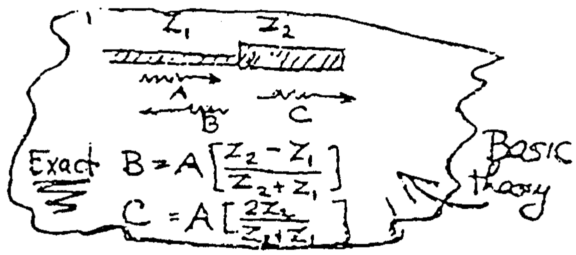
Reflections off Discontinuities

CASE I

Step discontinuity, as sketched:

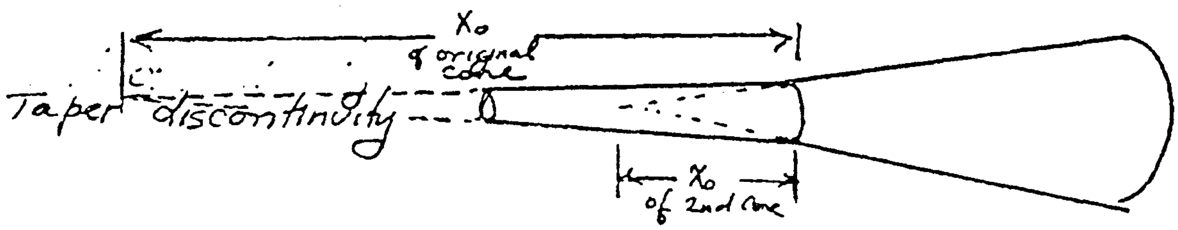
Fraction returned = $-\Delta a/a$
--INDEPENDENT of FREQUENCY.

This says the returned signal is an INVERTED COPY (reduced in size) of the incident signal from the player's mouth. Note: the return signal from the bell will eventually reach the discontinuity and itself be reflected in part back toward the bell. The sign of the returned fraction going toward the bell is opposite to the fraction returned to the mouthpiece.



Ordinary tuning slide discontinuities give $\Delta a/a \approx$ [from 0.04 (tuba, etc.) to 0.08 (trumpet, etc.)].

CASE II



Fraction returned =

$$\left[(x_o{}^{2nd} - x_o{}^{1st})/\sqrt{(x_o{}^{2nd} + x_o{}^{1st})^2 + (4\pi f/c)^2 (x_o{}^{2nd} \cdot x_o{}^{1st})^2}\right]$$

Depends on frequency

(a) at "low frequency"--x_o's are both much less than a half-hump long,

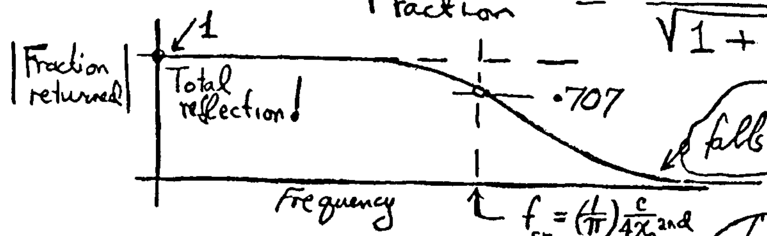
$$\text{Fraction} = (x_o^{2nd} - x_o^{1st})/(x_o^{2nd} + x_o^{1st})$$

(b) at "high frequency"--x_o's are both much longer than a half hump,

$$\text{Fraction} = [(x_o^{2nd} - x_o^{1st})/(x_o^{2nd} \cdot x_o^{1st})](c/4\pi f)$$

(c) Suppose the first pipe is nontapered ($x_o^{1st} \longrightarrow \infty$). Then the fraction returned is

$$\text{Fraction} = \frac{-1}{\sqrt{1 + \left(\frac{4\pi f}{c} x_o^{2nd}\right)^2}}$$



falls as $\frac{1}{f}$ above this critical frequency $= (\frac{1}{\pi})$ 1st mode freq of pipe of length x_o.

for $x_o^{2nd} \approx 1$ meter $f_{cr} \approx 30$Hz

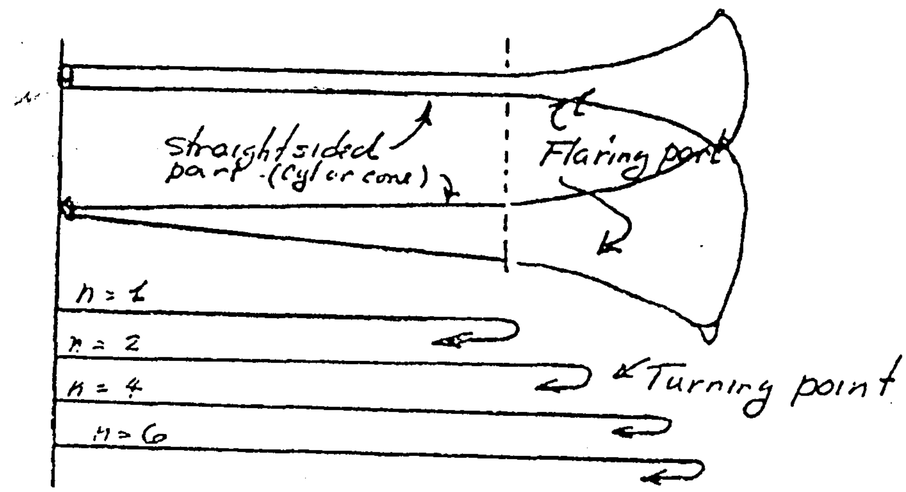
$$\frac{B}{A} = \frac{x_o^{2nd} - x_o^{1st}}{(x_o^{2nd} + x_o^{1st}) + 2j\left(\frac{\omega}{c}\right)(x_o^{2nd} \cdot x_o^{1st})}$$

this is exact - note phase shift also

Q. MISCELLANEOUS BRASS-INSTRUMENT INFLUENCES--PART II

Phase and Group Velocity Effects

Consider a typical brass-instrument air column (neglecting the mouthpiece):



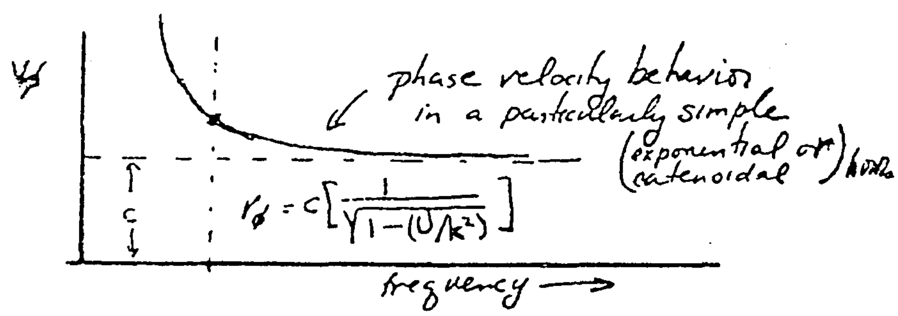
(Note! We are presuming that no garbage-reflections off of bore discontinuities are taking place.)

A wave launched at the small end has a "turning point" deeper into the bell for high-frequency sounds than for low-frequency sounds. This automatically makes the round-trip time somewhat longer for the high-frequency sounds than for the low, since all frequencies propagate with equal speed ($c \cong 345$ m/sec) in the straight-sided part of the bell.

It can be shown that the nth resonance frequency of the horn is a frequency for which TWO ROUND-TRIP TIMES is equal to n periods of the oscillation:

$$T_{round\ trip} = (1/2)(n/f_n)$$

Here we must take as the speed of sound c in the straight-sided part, and an increased value v_ϕ--the phase velocity--in the flaring part (there is some complication at the turning point, but it is shared by all round trips, so we can set it aside).



This is one way to specify the proper horn shape for good steady-state maintenance of oscillation--the round trip times must give harmonically related f_n's.

Phase Velocity and Round-Trip Distance Effects

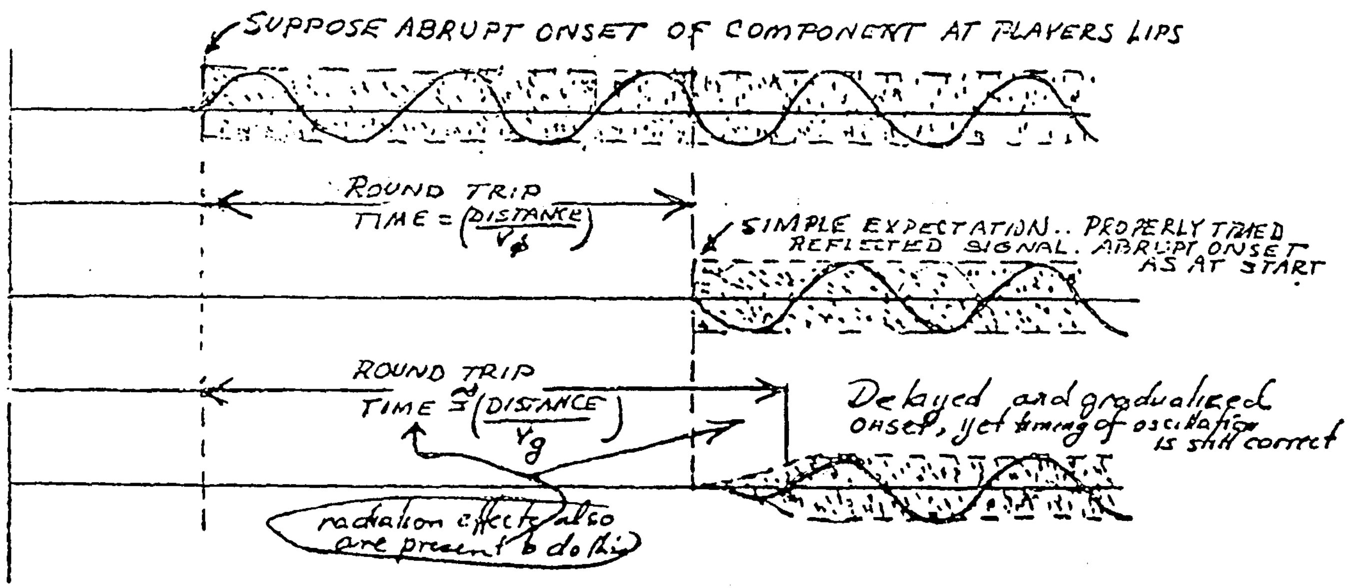
The implication of the foregoing discussion of the round-trip time is that when a horn player (for instance) starts his lips buzzing, he launches a collection of harmonically related sinusoids down the horn toward the bell in the expectation (hope?) that they will come back all in step to tell his lips that all is well, and so aid and abet the setting-up of a regime of oscillation.

So far we notice that for any attempt to start the note, the fundamental component will come back first to help the oscillation, and one by one the other, higher harmonics will trickle in to join the game. Thus, the security with which the enterprise begins is very low (before the fundamental component gets back) and then grows step by step as the various components come back in sequence. Note that the fundamental component MAY NOT be the strongest of the collection of initiating sinusoids--what does this imply?

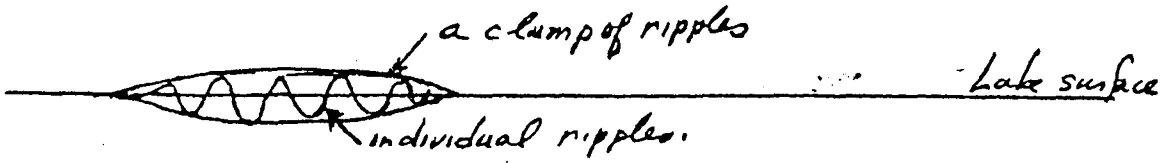
Group Velocity and Round-Trip Distance in Bell Effects

Something new comes in when we focus our attention on the higher-frequency components that might exist in a horn--say the 2nd, 3rd, and 4th harmonic components of a tone that is set up in steady state by the collaboration of peaks 4, 8, 12, and 16 of the horn (analogous to C5 on a trumpet).

All these upper components traverse nearly the whole of the length of the flaring bell of the instrument, where they are subjected to a new influence arising from the bell flare. EVEN IF THE SINUSOID RETURNS PROPERLY IN STEP, its buildup is delayed and gradualized.

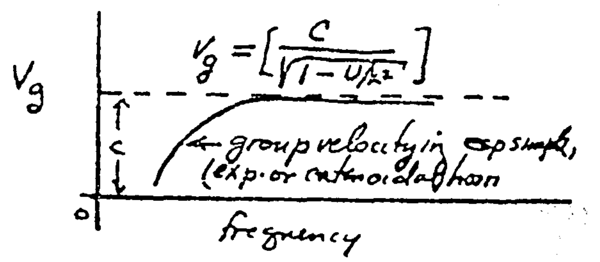


What causes this? The phenomenon known to wave physics as dispersion is one contributor. Look at a familiar example--ripples on a lake:



We observe that the clump moves across the lake at one speed, while the individual ripples move at another speed. It happens that for water ripples v_{ripple} (the phase velocity of v_ϕ) is greater than v_{clump} (the group velocity v_g), so the ripples appear to be born at the trailing end of the clump, move forward through it, and disapper at its front end. That is, $v_\phi > v_g$ for water ripples.

In outward-flaring horns, however, $v_g < v_\phi$.



The preferential leaking-out of high-frequency sound from the bell also has an (indirect) way of delaying and rounding the onset.

Closer inspection shows always that low-frequency waves have slower v_{group} than do higher-frequency waves in horns.

Think about the implications of all this phase and group velocity stuff on the relative security of startup of low and high notes on a modern and a baroque trumpet. The latter has a great deal of cylindrical pipe in it and a bell very similar to that of the modern instrument. How about the valve and natural horn?

Put all the BRASS INSTRUMENT INFLUENCES into your cogitational grinder and see why a good brass instrument has two easy tests of its quality:

(a) It emits a nice, clear-cut "pop" from the bell when a hand is slapped lightly onto the mouthpiece rim, to close it abruptly.

(b) It does not drift sharp or flat on a crescendo while playing C4, G4, etc.

IMPORTANT: Why is all this stuff not very applicable to woodwinds??

R. ASSIGNMENT 5 AND OTHER MATTERS

(1) Make a topical and logical summary of the course, consisting of a number of paragraphs, each with its own heading, to show the subject matter we have covered and the way in which it fits together in retrospect. (The order of presentation and the choice of particular instruments to illustrate general points was [for me] a matter of pedagogy rather than of absolute logical necessity once all the pieces were in hand.) This summary should not be longer than 3 or 4 pages, but I doubt that anyone can do an adequate job in less than 2.

(2) Write a brief sketch of the musical (tonal, technical, etc.) strengths and weaknesses of one woodwind and of one brass instrument over the past 250 years (less for the clarinet, and one need not go all the way back for certain other instruments). To give shape to these two outlines, use today's instrument as the point of reference. Helpful books will be those by Philip Bate on the oboe, flute, trumpet, and trombone; Geoffrey Rendall and Oskar Kroll on the clarinet (Bate's revision of Rendall is the best); Robin Gregory on the horn. Baines is, of course, your best main reference for the woodwinds! I will be glad to make suggestions to you individually, and/or negotiate minor modifications to the assigned task as may seem advisable.

Other Matters

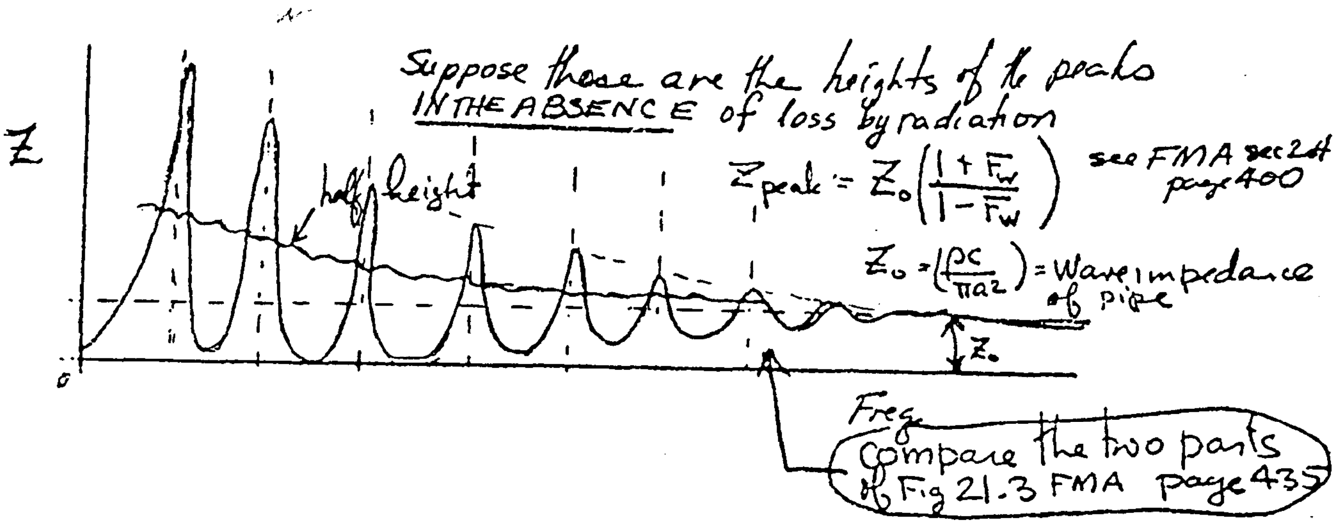
(1) A term paper subject must be settled with me by November 28, afternoon.

(2) The term paper should be in by December 16, and MUST be in by the morning of December 19.

(3) Whether or not we play for other people, we will have a session in which we play and show. By December 5 the details should be settled by each of us.

S. WALL MATERIAL, ETC.

For components of the tone of <u>any</u> instrument, more than 99% of the input energy from the player ends up simply heating the walls of the air column (below cutoff). Above cutoff, the fraction of the energy that is absorbed by the pipe walls falls. In a musical instrument it never reaches less than a 50-percent share in the immediate neighborhood of f_c. (See FMA, sec. 22.7, p. 499ff.)



Here F_w is the round-trip fractional amplitude if only wall losses are present. Near cutoff, but just below it, Z_{peak} may (for instance) fall to 1/2 the original value because of radiation losses. If we define F_{rw} as the round trip . . . etc., etc., . . . including wall and radiation for the special case where z_{peak} is 1/2 Z_p(original), then $F_{rw} = [(3F_w - 1)/(3 - F_w)]$ and the fraction of energy in the total input going to the walls is

$$\epsilon = \{[1 - (F_w)^2]/[1 - (F_{rw})^2]\}$$

Some examples:

F_w = 0.95 so that Z_{pk}/Z_o = 39 (as on a Boehm flute)

F_{rw} = 0.90 and ϵ = 0.53 . Recall $f_c \gtrsim$ 2 kHz for this instrument.

F_w = 0.90 so Z_{pk}/Z_o = 19 (as near D4 of the clarinet)

F_{rw} = 0.81 and ϵ = 0.55 This might be near 1100 or 1200 Hz, since $f \cong$ 1500 Hz.

F_w = 0.80 so Z_{pk}/Z_o = 9

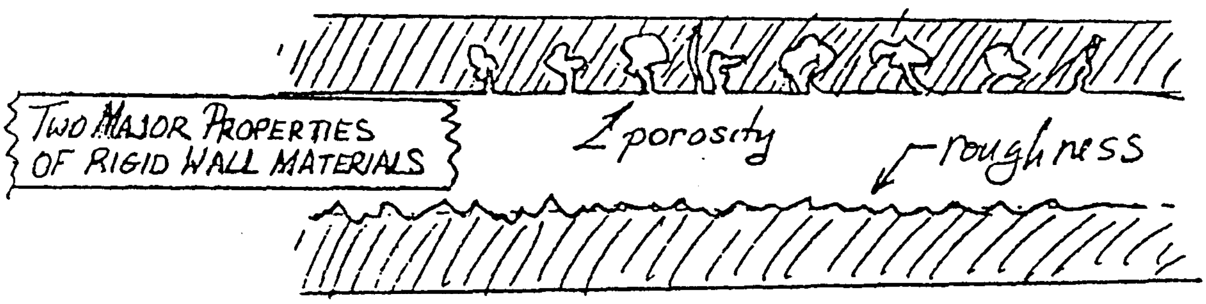
F_{rw} = 0.64 and ϵ = 0.60

F_w = 0.70 so Z_{pk}/Z_0 = 5.7 , as on lower res of a trumpet (see FMA, sec. 20.4, p. 400).

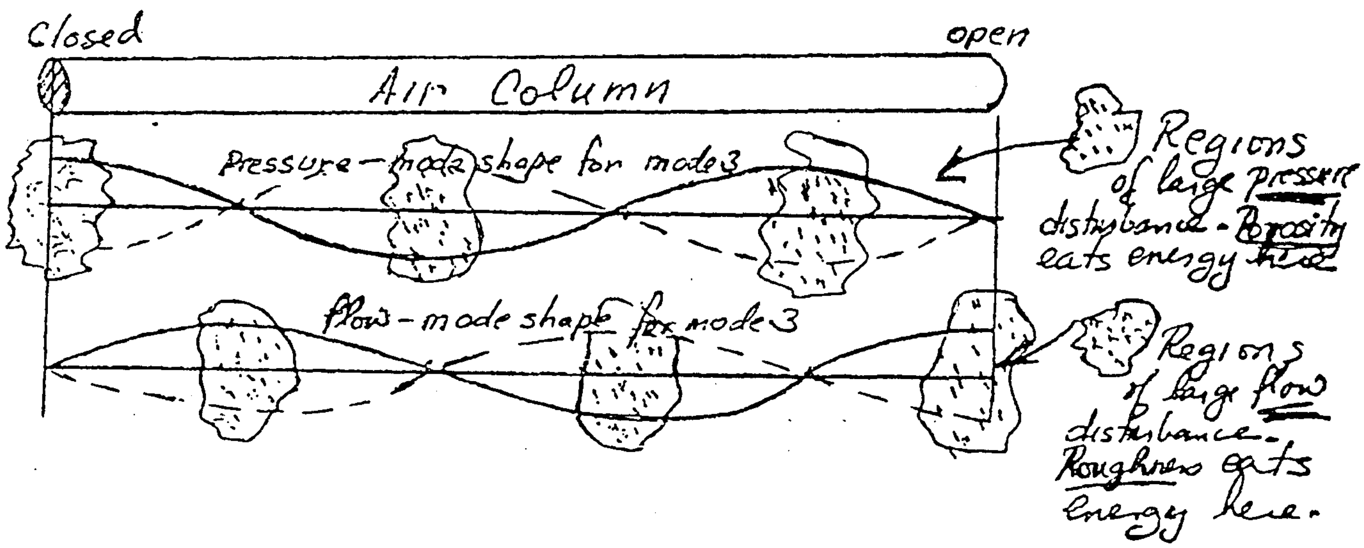
F_{rw} = 0.47 and ϵ = 0.66

Recall: Most of the energy production comes below f_c so we have <u>most</u> of the energy dissipated at the walls for all instruments.

All this goes to show that the material from which the instrument is made may be expected to play an important role in its playing behavior from energy considerations alone. There is of course much more to take into account.



Consider a standing wave in some air column:



Note: Given p(x,t) at the frequency ω,

$$u(x,t) = (\pi a^2/j\omega\rho)[(-\partial/\partial x)p(x,t)]$$

DIFFERENT PARTS OF THE AIR COLUMN EAT ENERGY IN DIFFERENT WAYS--A GIVEN SEGMENT OF THE BORE BEHAVES DIFFERENTLY FOR DIFFERENT MODES!!

Very important and useful: Notice that the toneholes and their slightly yielding, porous pads are themselves contributors to the energy loss.

Even for absolutely smooth and rigid, nonporous walls, there is some energy loss, viscous (as before but smooth-walled) and thermal (behaves like porosity, because temperature rises and falls with pressure--walls serve to absorb and give off heat).

ALL WALLS: Viscous loss (rough or smooth) plus thermal and porous losses.

The thermal part of the dissipation varies only slightly with wall material. As compared with a perfectly conducting material of infinite heat capacity and density, we have

copper	0.9997
brass	0.9994
German silver	0.9991
wood*	0.9720

*All woods turn out pretty much the same. Notice that wood has LESS thermal energy absorption than does metal!

Experiment shows that a player can just about (usually) detect a change of about 2%, so that wood vs. metal is just detectable. This is NOT to say that the listener can detect the change.

Because the porosity of wood is greater than metal, actually the porosity/thermal (i.e., pressure-associated) losses in wood are somewhat greater than in metal, 2 ⟷ 10%, depending on surface condition and material. It can go higher yet, but then the musician complains seriously. (The porosity referred to here includes the porosity of the pads, etc.)

Now look at viscous (flow-associated) dissipation.

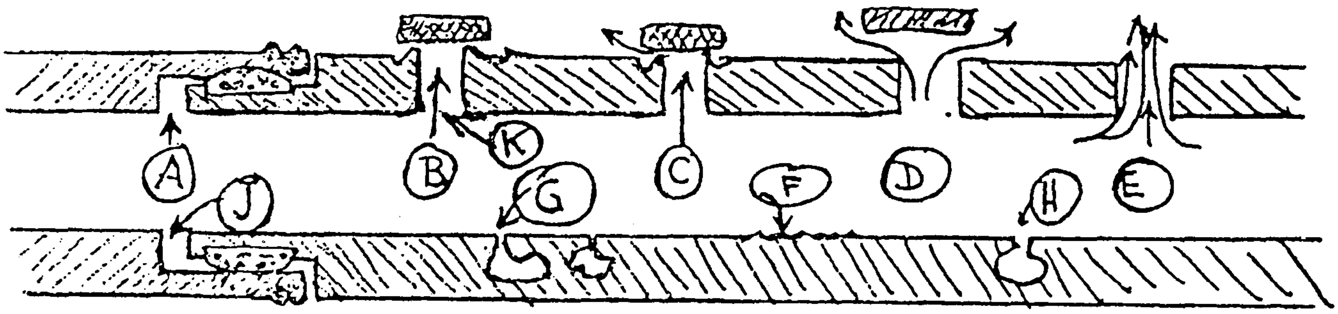
For polished wood (Grenadilla, etc.) and metal at low sound levels, the losses are pretty close to those for infinitely smooth surfaces. As long as the roughness ranges over no more than ± 0.05 mm at 250 Hz (this doubles for every 2 octaves you go down), the surface acts smooth.

At very high sound levels, e.g., in the backbore of a trumpet mouthpiece at fff levels, there is evidence that the smoothness of the surface must be much improved if there is to be no added dissipation. More on high-level effects in due course.

For idealized surfaces the viscous loss is about 84% of the thermal loss. Both rise as $\sqrt{\omega}$.

We are not quite ready to look at the musical implications of all this yet, but we should collect the information as it relates to the structure of musical instruments.

Consider a woodwind joint:



(A) If the joint is in a high-pressure region of the standing wave, it can eat a lot of energy, especially in spot where the gap < 0.1 mm, which doubles the thirst for energy.

(B) The added surface area of the tonehole plus the yielding aspect and porosity of the pad can eat pressure-type energy.

(C) A slight leak under a pad also steals energy if it lies in a high-pressure region of the standing wave.

(G) Porosity in the wood is another cause of dissipation if it lies in high-pressure-variation regions of the bore.

(D) and (E) There are strong viscous losses in the open toneholes, especially if there is a pad too close. There is a great deal of flow in the first open tonehole.

(F) and (J) Rough surfaces also eat energy from the flow aspect of the standing wave. Shortening the gap at a joint helps this, but spoils (A)!!!

(K) Flow past a closed tonehole can give rise to a little extra viscous dissipation.

(H) Porosity does not play much of a role in the high-flow regions of a standing wave.

USEFUL! IMPORTANT! A GENERAL REMARK: the W-curves for bore perturbations give a quantitative measure of the viscous and porosity-type losses. Where W is positive, pressure losses predominate. Where W is negative, viscous ones predominate.

T. WALL MATERIAL, ETC., FOR THE PLAYER

Review Some Basic Information

(1) Energy input to a wind instrument is almost totally spent within it.

(2) This is a good thing, or room effects could disrupt the playing.

(3) Wall effects: Primary energy absorption to first approximation

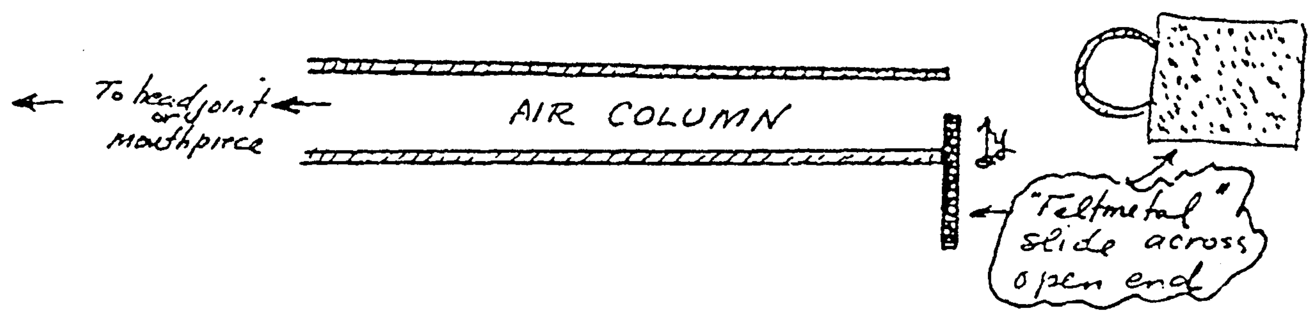
(a) Porosity and thermal conduction (absorbs from the PRESSURE aspect of the vibration). Porosity conduction is slightly larger than thermal.

(b) Viscous ($\approx$ same for all if polished) (absorbs from the FLOW aspect).

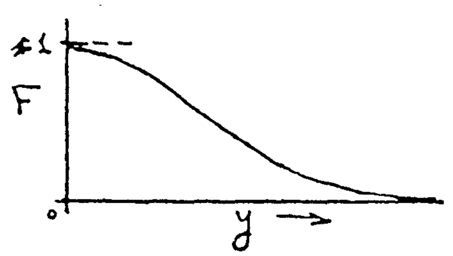
Side remark: The difference between wood and metal is at the ragged edge of detectability by the player IN LABORATORY TESTS! He is slightly more sensitive than direct measurement experiments in the physics lab.

How Does One Find This Out?

Basic experiment, using clarinet- or flute-type excitation:



Feltmetal is chosen because it is essentially nonreflecting when it totally covers the end of the tube. If y represents (somehow) the amount of covering of the end of the tube, the round-trip attentuation F varies with y, thus:



BUT, THE RESONANCE FREQUENCIES ARE NOT ALTERED APPRECIABLY (the effect amounts to only a few cents).

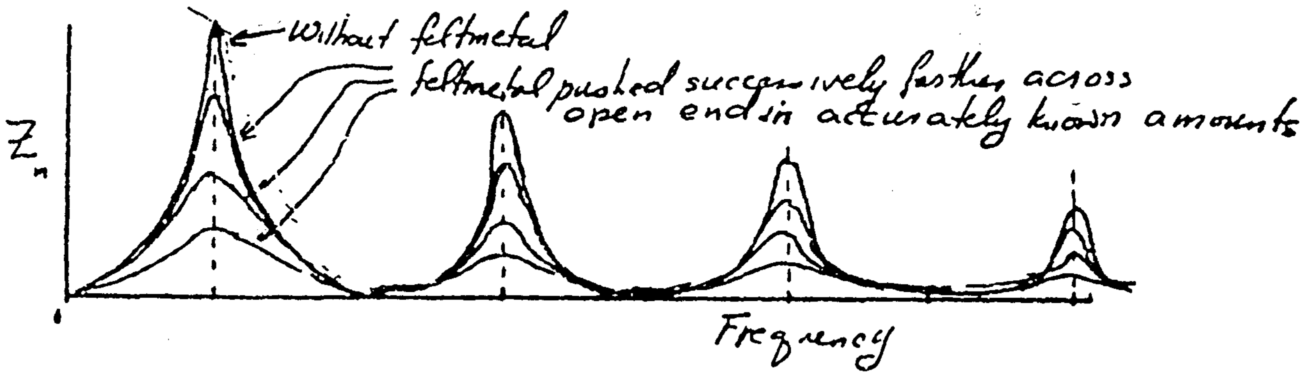
Recall peak height,

$$Z = [(c)/(\pi a^2)][(1 + F)/(1 - F)]$$

$$Y = [(\pi a^2)/(c)][(1 + F)/(1 - F)]$$

The mathematics of this feltmetal termination is rather interesting!

In the lab we measure an impedance curve for each of several well-defined settings of the feltmetal:

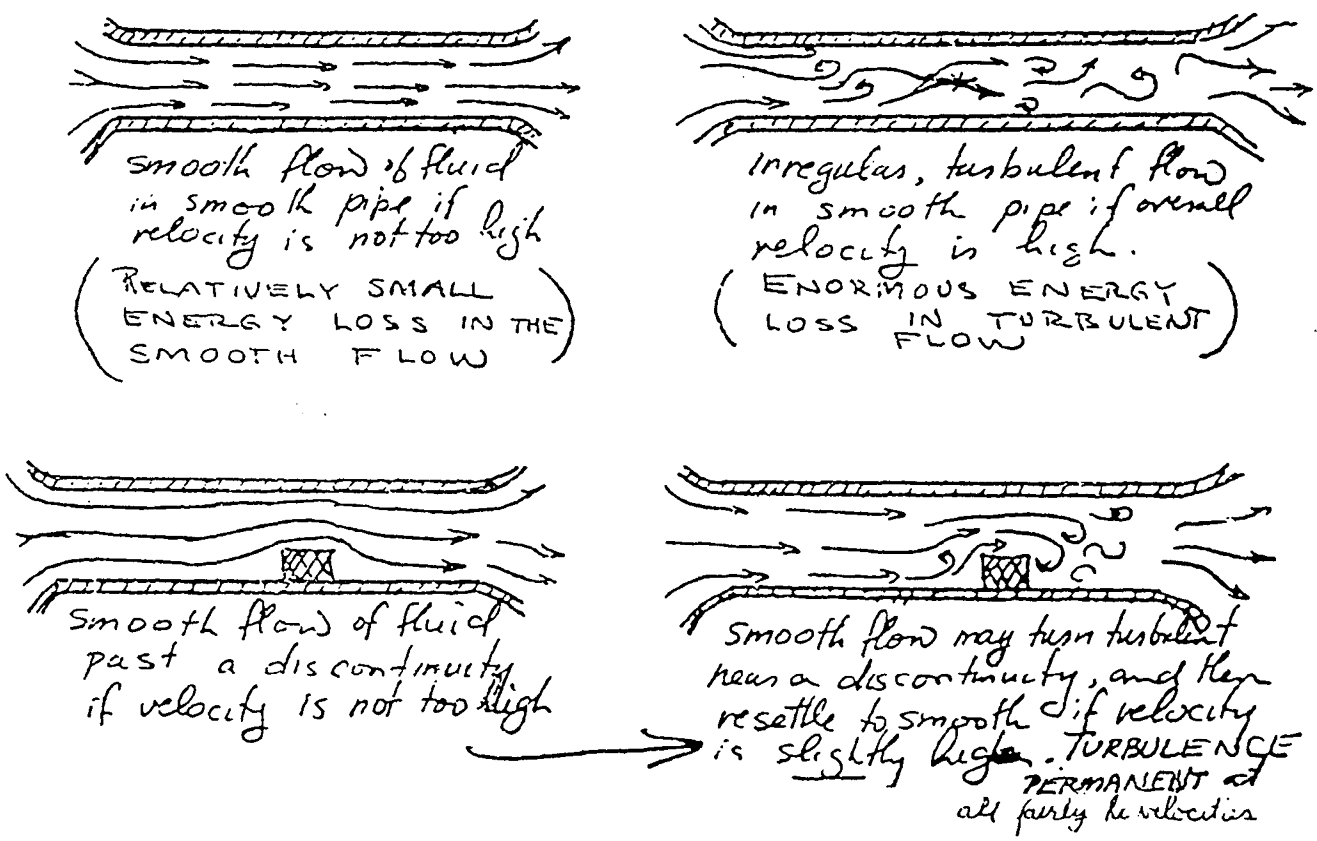


These curves give accurately known values for the damping, and it is a simple matter then to interpolate between them to estimate the player's sensitivity to small changes in the position of the feltmetal. Changes of 1 or 2 percent are detectable by the player.

For air, $\eta \approx \sqrt{Rm\rho}$; $\rho \approx 1/\text{Temp}$; $\eta/\rho \approx T^{3/2}$ (the flute plays better in hot weather--when we discuss turbulence, we will find out why).

ALL THIS IS FINE, BUT BIGGER EFFECTS ARE PRESENT--TURBULENCE

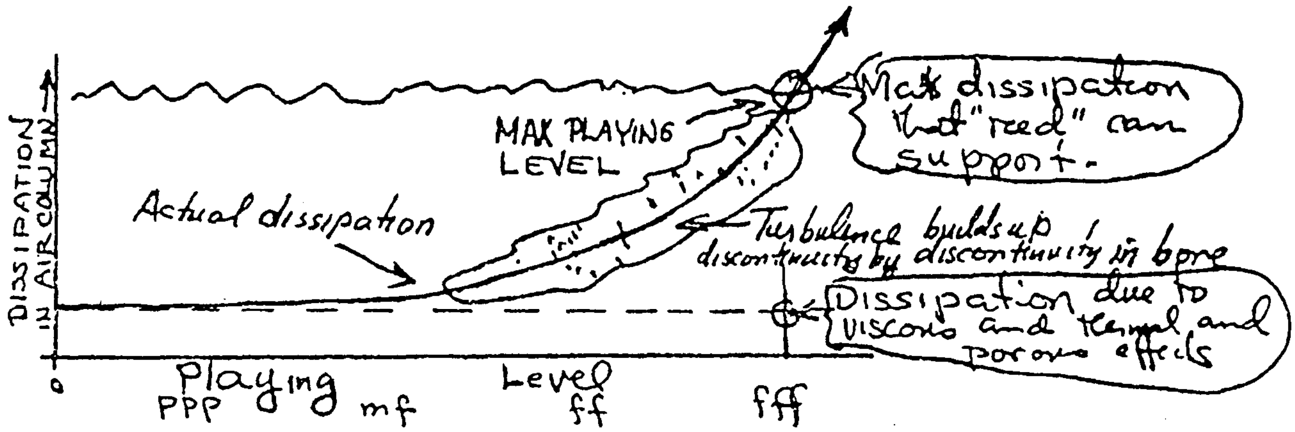
This is introduced via steady fluid flow rathen oscillatory:



The turbulence onset velocity is dependent on kinematis viscosity (η/ρ) (η is the viscosity coefficient, and ρ is the density). High kinematic viscosity leads to high velocity before turbulence breaks in.

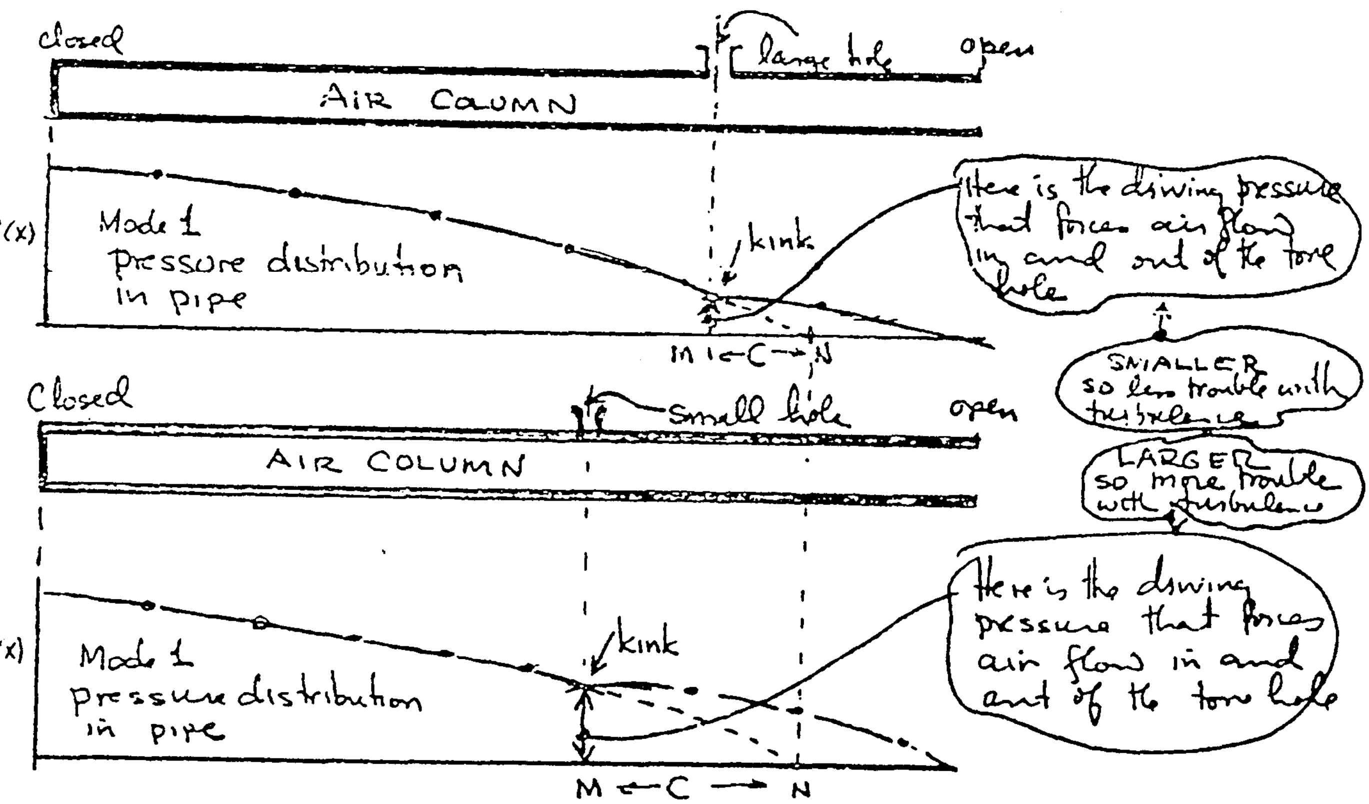
VERY SIMILAR EFFECTS OCCUR IN AN OSCILLATORY FLOW (hissing at the first open tonehole, for instance).

Recall that raising the dissipation within an air column lowers the resonance peak heights via changes in F. If dissipation gets greater than what the reed can support, there is NO OSCILLATION.



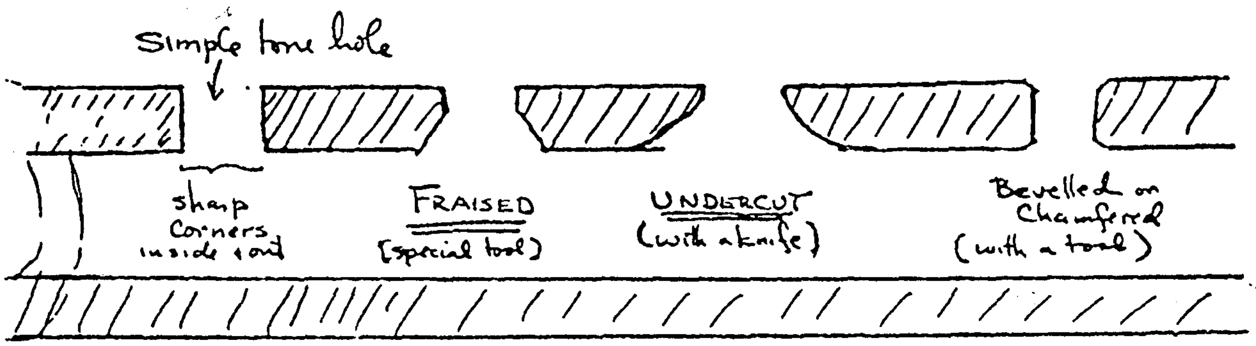
BIG vs. SMALL HOLES

For the same frequency on an air column, a big hole is farther south than a small hole (see FMA, Fig. 21.10, p. 454).



The increased tendency for turbulence troubles on the part of small toneholes was an important driving force for the Theobald Boehms of the nineteenth century. "Better" technology led to sharper corners in the carpentry, which forces larger holes and/or "better venting." (See FMA, sec. 22.7, p. 499ff.)

Let us look at some kinds of toneholes and assess their sensitivity to turbulence troubles.



We need somewhat stricter terminology than is customary. What I have here is approximately what is usually given, but the categories are distinct.

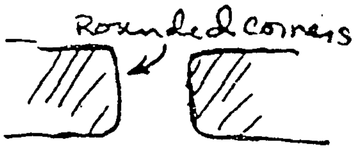
Fraised--much less turbulence trouble for closed hole, somewhat less when open.

Undercut very much--very much less trouble for closed hole; considerably less when open.

Bevelled--slightly less in both cases.

If corners have sharpness such that the radius of curvature is more than $0.1 \times \sqrt{250/f}$ mm at the lowest note of the instrument, things won't give too much trouble, therefore use larger radii if at all possible.

WITH REALLY SHARP CORNERS, MICROSCOPIC CHANGES ARE READILY DETECTED BY THE PLAYER (e.g., 10 sec thumb-wear on a brand new grenadilla clarinet tonehole)!!!

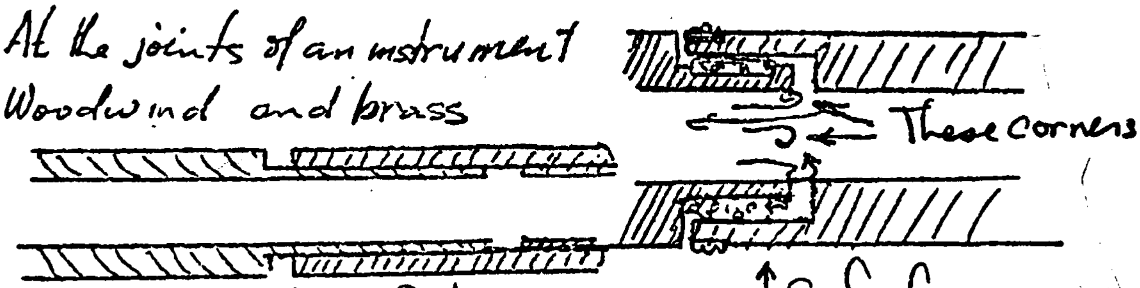


THESE MICROSCOPIC CHANGES DOMINATE the musician's recognition of the differences between one instrument and its supposedly identical twin, in the case of today's machine-made clarinets, flutes, oboes, trumpets, saxophones, bassoons.

Let us list a few spots where sharp corners produce musically significant turbulence effects that alter the "feel," dynamic range, cleanness of initial startup, and clearness of established tone.

———→ NOTE ALSO THE WARNINGS!!! ←———

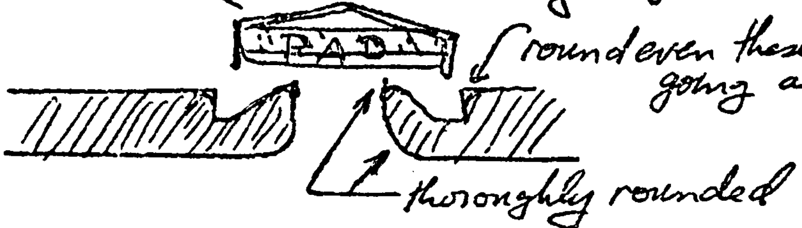
① At the joints of an instrument
Woodwind and brass



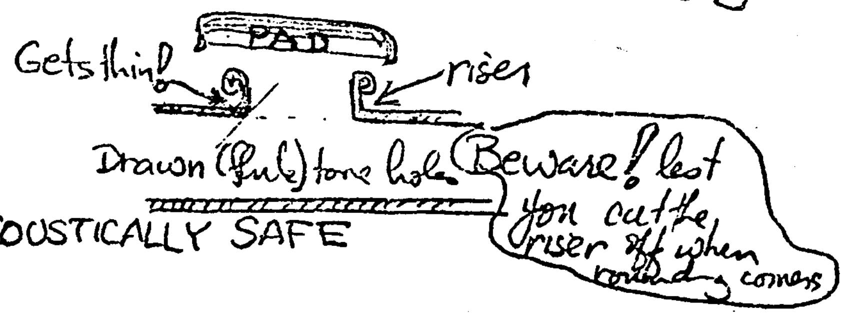
Brasses: not too safe because of the Terrible Consequences of getting grit, chips + abrasive into the valves rotors and slides. Acoustically safe however

↑ Safe for anyone to round these (mechanically + acoustically safe)

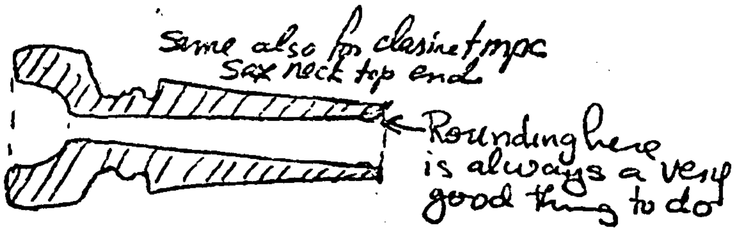
② Tone holes (and waterkeys of brass instruments).



NOT SAFE ON WOODWIND UNLESS YOU CAN REALIGN TONEHOLE SIZING FOR BEST RESPONSE--very subtle business



ACOUSTICALLY SAFE



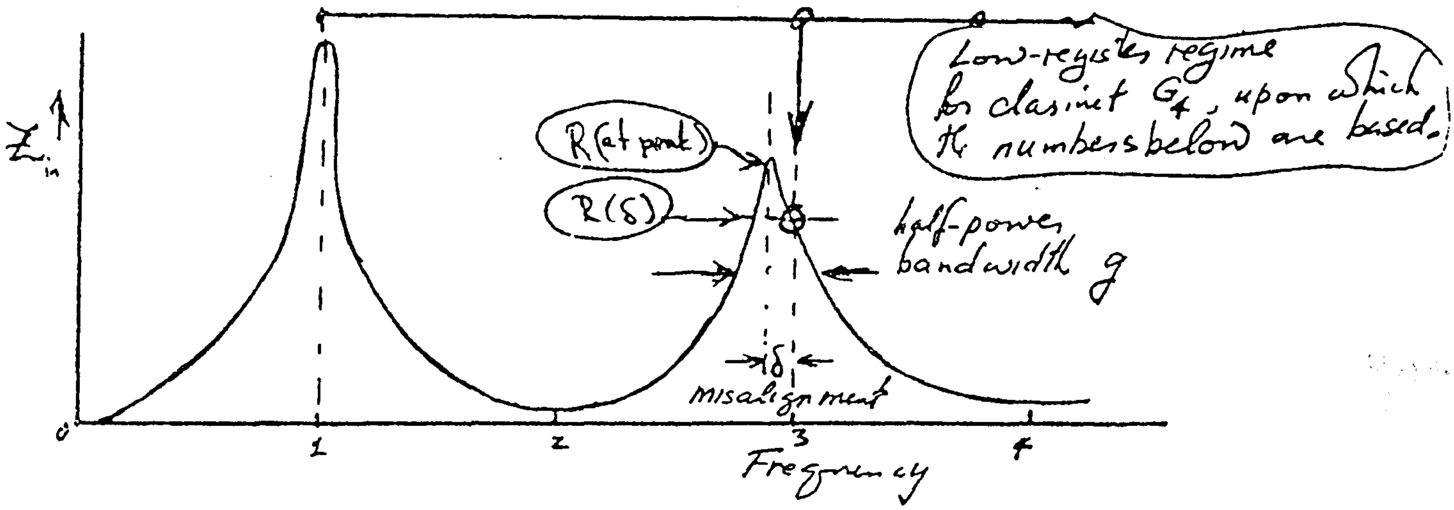
NOTE! Never ream and paint the bore and toneholes unless the corners are rounded first: players will always distort their playing note by note to minimize turbulence effects so that centering and tuning errors sometimes are caused by "corner-itis." This should not be corrected by perturbations to the basic layout--remove the touchy little creatures first, then see what are the demands of the real acoustics of the system.

U. FURTHER CONSEQUENCES OF WALL LOSSES

Setting aside the turbulence phenomena (which are very important in all real instruments) outlined in the previous section, let us look some more at the musical consequences of damping in a wind instrument.

Observation 1

It is an experimental fact that one must align the cooperating resonances within about 10 cents if the player is to be made really happy (i.e., he can detect misalignments greater than this without formal experimentation). Let us see what this implies:



Consider a clarinet. How much is the height of the [real part] of Z_{in} reduced for peak 2 if it is our of line by an amount δ/f_3 --> 10 cents ≡ 0.006, given that the half-power bandwidth $g \cong (f_1/30) \times \sqrt{3} = 0.06$?

$$R(\delta) \cong (\text{const}/g)\{1/[1 + 4(\delta/g)^2]\}$$

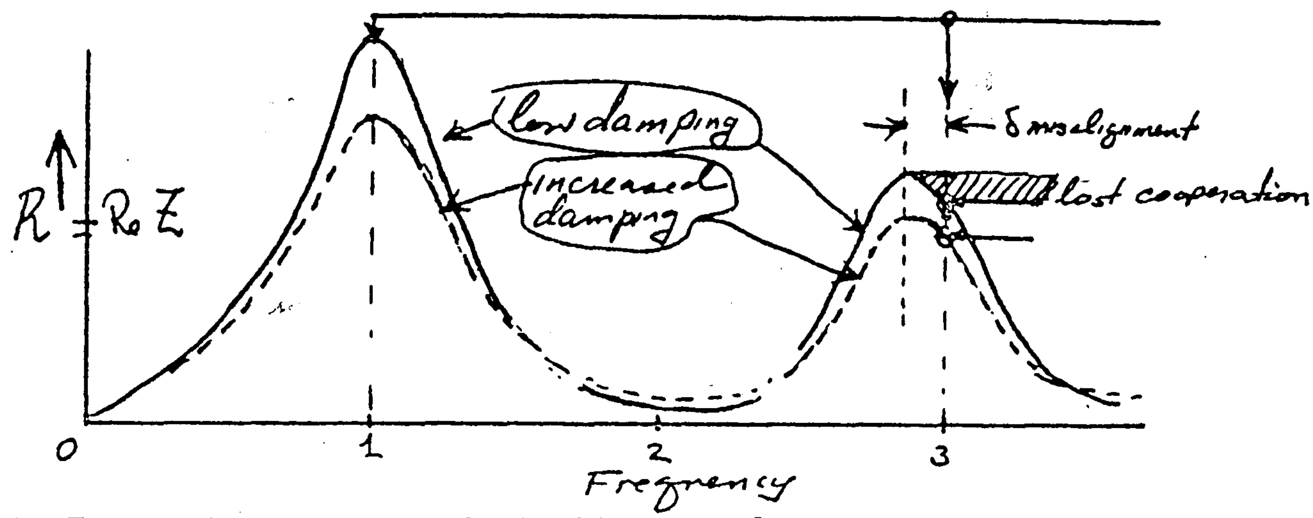
When $(\delta/g) = 0.1$ then R(10 cents off) = 0.96 x R(exact)

Thus, a change of <u>less</u> than 4% in the energy production is detectable by the player. Recall that <u>MOST</u> of the energy is produced via peak 1 so that a 4% change in the peak-2 contribution is a very small share of the whole.

Observation 2

A mildly out-of-alignment instrument will usually play better if porous pads are used on it, or if the walls are left unoiled. Reducing the damping may make it easier to blow, but the steady, clean responsiveness of a properly set-up instrument will be lost. How can this be understood? What does it imply for the physics of the player's situation?

Let us focus our attention on the second of the cooperating resonance peaks, without however forgetting about the first one:



(a) For a given amount of misalignment δ and a fairly low damping, we can see from the solid curves that there is a certain amount of lost cooperation at peak 2 (shown by the cross hatching extending beyond the peak).

(b) Increasing the damping on this instrument (with misalignment kept constant) reduces the heights of peak 1 and peak 2 by the same percentage amount. It also increases the bandwidth of each resonance--by this same percentage amount.

(c) It is the magnitude of R that really tells us what happens when we change the damping. For given misalignment δ,

$$(dR/R) = -(dg/g)[1 - 8(\delta/g)^2]$$

for each mode. For mode 1, $\delta = 0$, so a 10% in damping ($dg/g = 0.10$) leads to a 10% change in the height of the peak. This will make the instrument feel stuffy.

For mode 2, $(\delta/g) \cong 0.012/0.06$, say (= 0.20),

$$(dR/R) = -(dg/g)[1 - 8 \times 0.04] = -(dg/g) \times 0.32$$

This says that one loses only about 1/3 (32%) of the mode-2 contribution associated with an increase in damping.

(d) We can now interpret the effects from a musician's point of view.

(1) Uniform loss of peak heights at the harmonics of the playing frequency merely makes the instrument play stuffy--one blows harder and tires more quickly, but does not consider the tone and response spoiled.

(2) The general quality of the instrument (its responsiveness, tone color, etc.) depends on the degree of cooperation between resonances. If we like, we can say that it depends heavily on the relative heights of the second/first resonance peaks (that is, the heights at the

frequencies of the played components!) Increasing the height of peak 2 relative to 1 makes a better instrument (as long as peak 1 is taller than peak 2!!!).

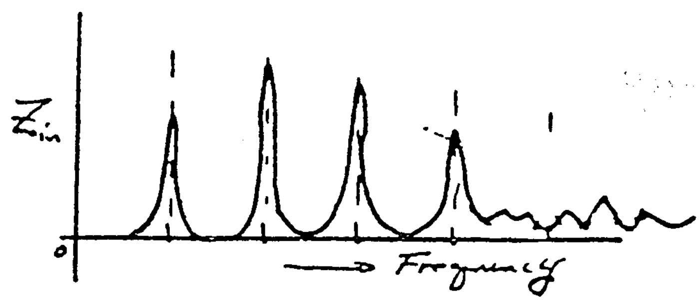
For the case sketched on p. 72 of these notes, the more heavily damped version is a better instrument, as long as fatigue is not a factor.

<u>Heavier damping within reason can be traded for mild misalignment</u>. Here is a spot where players have much experience, much folklore--mostly correct in essence though not always clearly stated.

Notice that the figure on p. 73 implies that progressively increasing the damping uniformly for both peaks of a slightly misaligned woodwind can cause a transition from register 1 to register 2--can you figure out UNDER WHAT CONDITIONS of reed resonance this will happen?

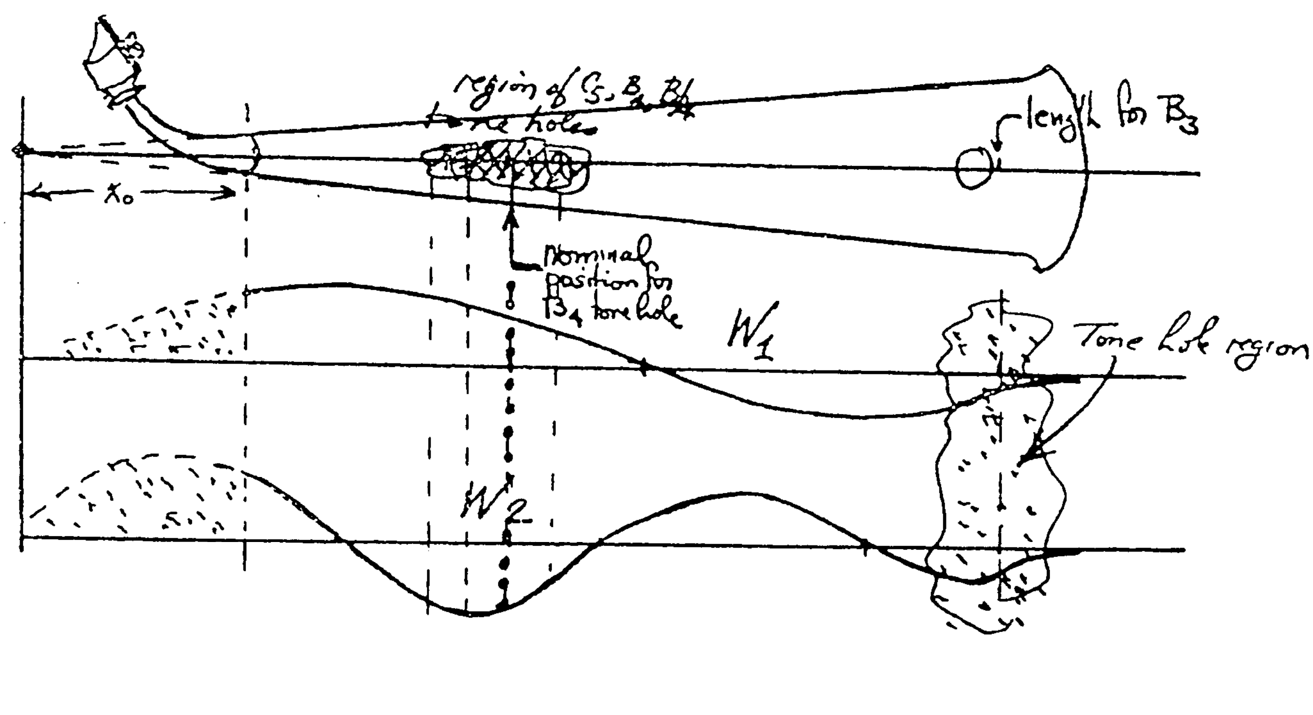
Let us now look at a practical example of the uses to which air-column "damping-ology" can be put.

We have repeatedly faced the fact that the bottom notes of a conical woodwind are blurty, coarse, and hard to control. The reason given was that peak 2 gets to be taller than peak 1 for a nearly complete cone. For example:



What can we do to reduce the height of peak 2 without lousing up peak 1 (and without spoiling too much of the behavior of the rest of the notes on the instrument)?

<u>Consider a straightened-out saxophone:</u>



Recall that where W-curves are above the axis, porosity eats energy, and where W-curves are below the axis, flow damping predominates.

(a) Ordinary sax pads are porous (or the horn gets harsh).

(b) Sealing the pores of the leather pads for the L-H fingers (C5# through B4-flat) reduces the damping of mode 1 and so raises the height of peak 1.

(c) This sealing does not do much to the damping of mode 2 since flow losses here come mostly from the tonehole edges, etc., etc.

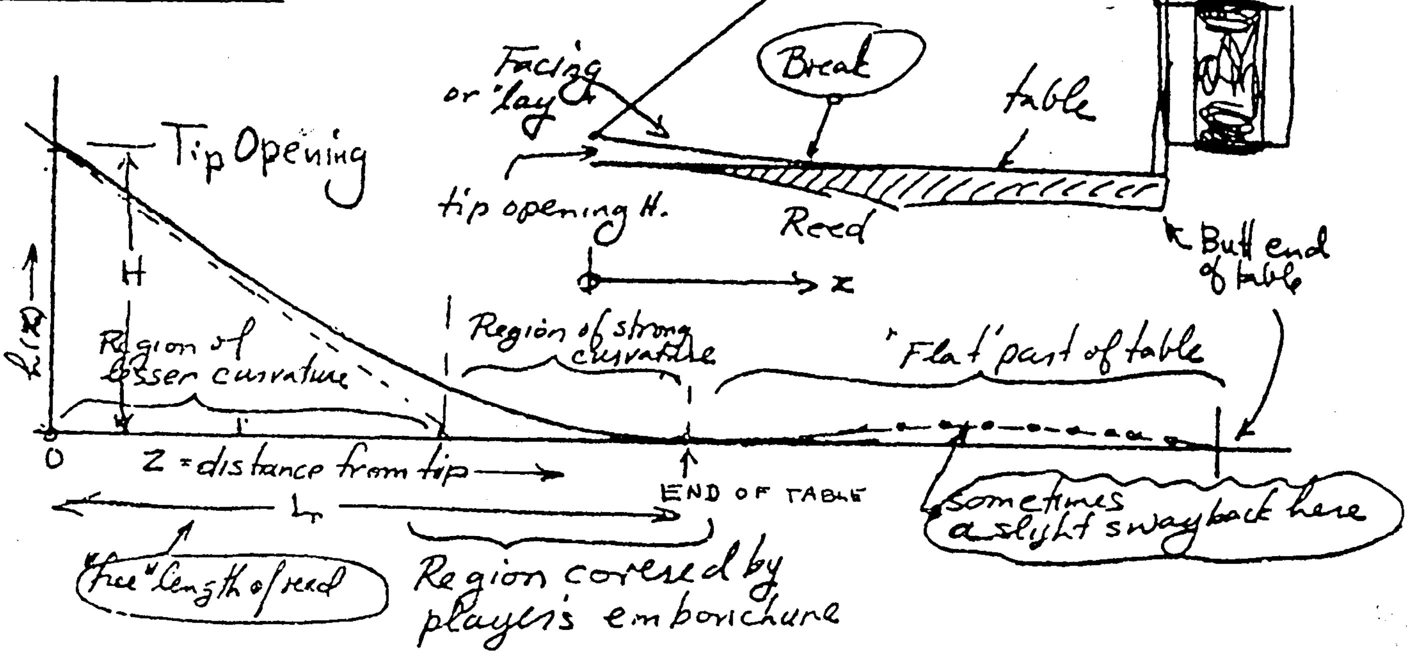
Explore the effects on other notes via W-curves in FMA, Fig. 22.4. Use F2 there where we have written B3; use F3 for the note B4 here. Or use the oboe perturbation curves that are in section J of these notes.

What do you suppose might usefully be done to help an oboe in the time [?] of its trouble (C4 down to B3-flat). How about roughening the bore as well as sealing the pads. Why <u>not</u>?

V. ON SINGLE REEDS AND MOUTHPIECES

(Note: In a general sense, most of this material applies also to the double reeds and; with some changes, to the brass player's lip reed.)

Basic Structure



Dominant requirements for the facing:

The region of strong curvature on a clarinet mouthpiece needs to cover about 1/3 of the distance from the end of the table to the mouthpiece tip. This lets the player vary the reed natural frequency over the 50% range (e.g., 2000 to 3000 Hz) needed for proper playing:

$f_r \cong const/L_r^{.85}$ for a clarinet reed, typically. If the thickness of the reed $t(z) = t_o(Z/L_r)^{\alpha}$, then $f_r = const/L^{\alpha-2}$. What about $\alpha = 2$!!!?

(Worman thesis, Appendix B, p.111)

The region of lesser curvature lets the reed progressively lie down on the facing as it rolls closed. If it is too straight, it claps shut abruptly on an attempted crescendo; if it is too curved at the tip, it leaks air and becomes hard-blowing without any compensatory benefit.

A word about reed profiles:

One can use either thick or thin reeds and still get the right range of f_r located in the right region--

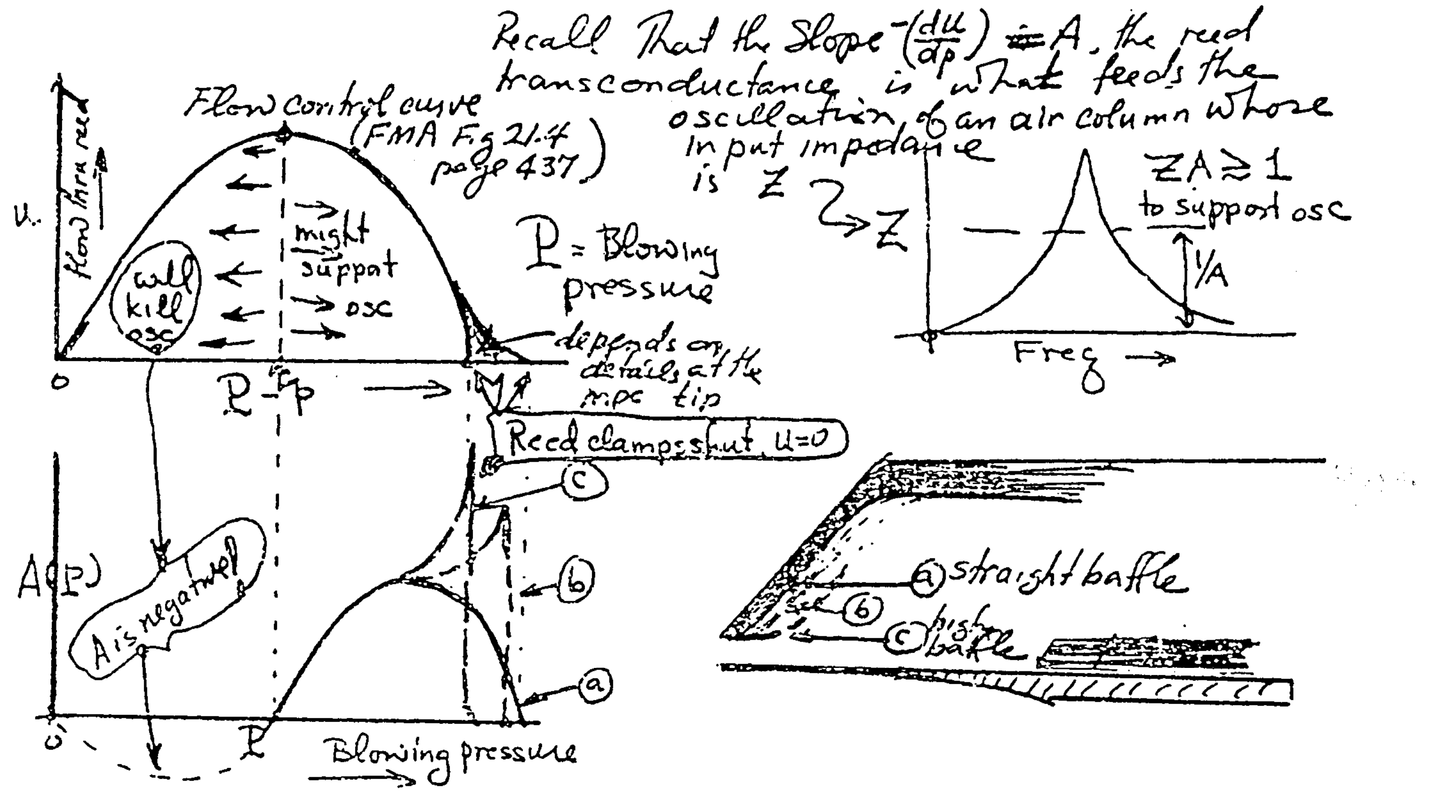
(1) f_r at any length $\approx$(1/basic thickness)^3; a thin reed leads to low freq.

(2) Thinning the tip raises f_r; thinning the root lowers f_r.

So--a thin reed is relatively thinner at its tip than is a thick one if it is to work on a given facing. (Sanding away on the flat side of a reed will produce this effect, since 0.001" at the tip is a bigger fraction of the original thickness at the tip than at the root.)

READ EVERYTHING ABOUT HISTORICAL AND NATIONAL VARIATIONS IN REED-MOUTHPIECE PROPORTIONING IN THE LIGHT OF THESE NOTES (Baines, Bate/Rendall, Kroll, or anywhere else)

Once we have the reed frequency located properly and properly controllable over the requisite range, we can look at its primary duty as a pressure-operated flow controller.



Straight baffle ⟶ small Bernoulli force at reed tip

High baffle ⟶ large Bernoulli force at reed tip

High baffle ⟶ reed tends to snap

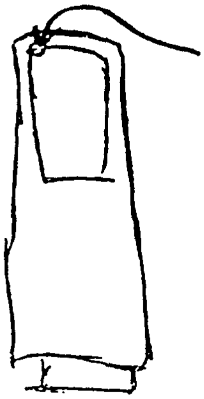
Shut when nearly closed ⟶ A is <u>very large just before reed closes</u>

REMEMBER: LARGE A ⟶ ability to support oscillation on a low Z peak

High baffle ⟶ reed screeching at almost any squiggly peak or taking off at its own frequency f .

High baffle ⟶ easy blowing (other things being equal), IF YOU CAN CONTROL IT (and if you like the much wilder tone)

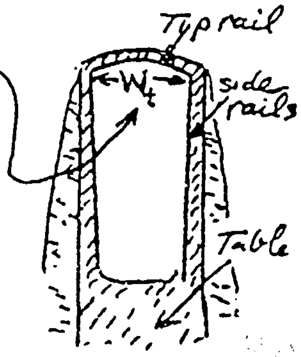
NOTE: MINUTE CHANGES MAKE BIG EFFECTS



1/2-mm square patch at one or both corners of a just visible high baffle on an otherwise straight baffle will convert a basically dull tone into a dark, smooth one with just a trace of flavor to it (if the instrument is in good alignment). Why does one get a wider dynamic range with a straight-baffle mouthpiece?

We will look now at some of the variables that control the overall magnitude of the transconductance A--given that its dependence on blowing pressure is always of the sort sketched on the previous page. Suppose we keep the same facing profile and baffle shape and also the same reed.

Alteration (P):



(a) Widening the aperture at the mouthpiece tip increases A proportionallay because the wider aperture at the reed tip for a given opening as it swings permits more airflow.

(b) Widening the aperture also increases A proportionally by increasing the area of the reed tip on which the mouthpiece pressure can act--so for a given reed stiffness we get more reed deflection for a given mouthpiece pressure.

CONCLUSION:

$$[(\Delta A)/A] = 2[(\Delta W_t)/W_t]$$

a 5% increase in tip width makes a 10% increase in A

Note: wider tip on mouthpiece → easier blowing with no loss of anything advantageous, as long as the reed overlaps the side rails enough that there is no leakage.

Suppose we keep the mouthpiece constant and change the width of the reed (the reed-tip width is W_r).

Alteration (Q):

(a) Widening the reed tip width increases the stiffness of the reed and so reduces A:

$$[(\Delta A)/A] = -[(\Delta W_r)/W_r]$$

(b) Increasing the width does not necessarily change the reed frequency f_r, but it does require greater embouchure pressure to pull the reed down the right amount on the facing.

Wider tip on reed → harder blowing with no loss of advantages.

We can use alterations (P) and (Q) to equalize the feel of mouthpieces of widely varying other properties--e.g., "matched" bright and dark mouthpieces via baffle height.

W. REEDS AND EMBOUCHURES (continued)

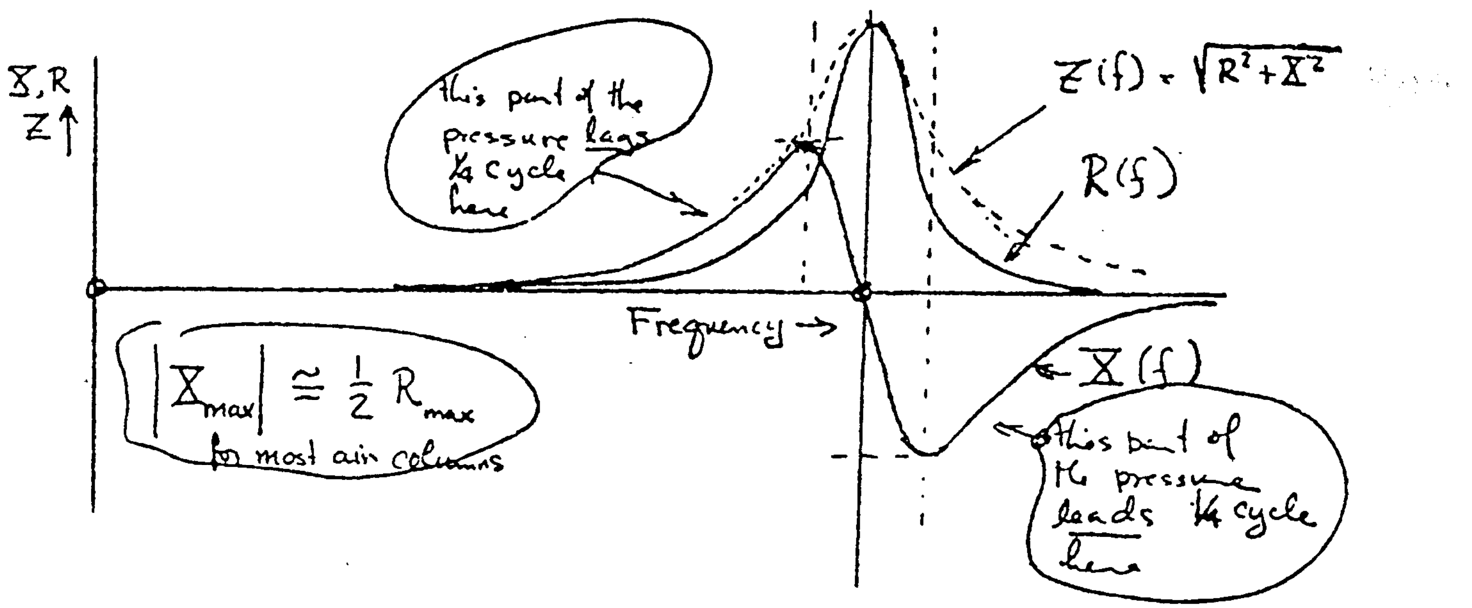
(These notes form the scientific background for class discussion of the effect of the player's lips on a reed, particularly in the case of double reeds. My adventures with blowing the tenora in Barcelona last summer will provide the anecdotal basis for the discussion, since they showed in exaggerated form much of what goes on in the playing of other instruments.)

(A) The impedance behavior of an air column in the neighborhood of a resonance peak is a little more complicated than I have admitted so far. There are two components to the pressure response of a pipe to a sinusoidal injection of air flow.

"in phase," "real": $R(f)$ = a component that swings <u>in step with</u> the excitory flow

"quadrature," "imaginary": $X(f)$ = a component that swings 1/4 cycle <u>out of step</u>

We have been discussing so far $Z(f) = \sqrt{R^2(f) + X^2(f)}$ because most phenomena have proved explainable on the basis of the net magnitude of Z up to this point:



The zero crossing of X does <u>not always coincide</u> with the peak of R, especially in brass instruments, via a mouthpiece effect.

For a cylindrical pipe open at far end

$$R = \left(\frac{\rho c}{\pi a^2}\right)\left[\frac{1+\tan^2 kL}{Q_0^2+\tan^2 kL}\right]Q_0, \quad X = j\left(\frac{\rho c}{\pi a^2}\right)\left[\frac{\tan kL}{Q_0^2+\tan^2 kL}\right]\left[Q_0^2-1\right]$$

Where $Q_0 = \frac{F+1}{F-1}$, F the relative ampl. after 1 round trip in the pipe

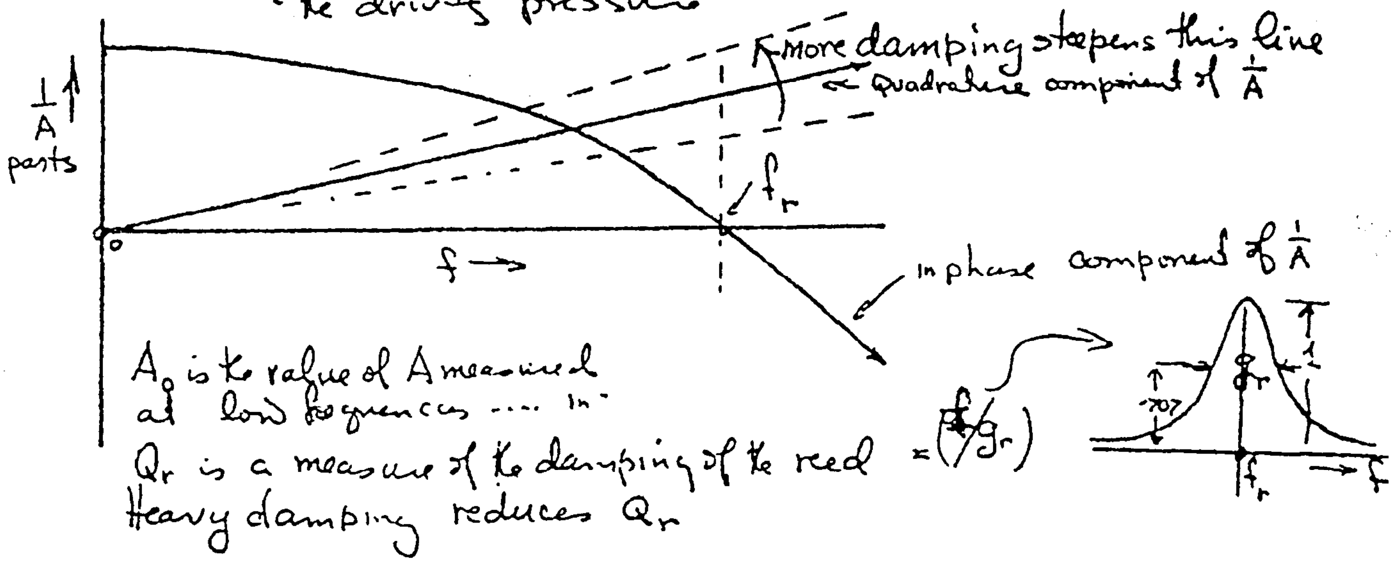
(B) Recall that to feed energy into an oscillation, the reed's flow control characteristic A has to be properly related to the air column. We had

$$ZA \gtrsim 1 \quad \longrightarrow \quad Z \geq (1/A)$$

In our closer look at things, remembering that reeds also show resonance behavior, we find that the coefficient (1/A) itself has an "in phase" and a quadrature part:

$\left(\frac{1}{A}\right)$ aspect { Component of controlled flow that is in phase with the reed driving pressure $= \frac{1}{A_o}\left[1 - (f/f_r)^2\right]$

$\frac{1}{A}$ aspect { Component of controlled flow that is 1/4 cycle out of phase with the driving pressure $= \left(\frac{1}{A_o}\right)\left(\frac{1}{Q_r}\right)\left(\frac{f}{f_r}\right)$



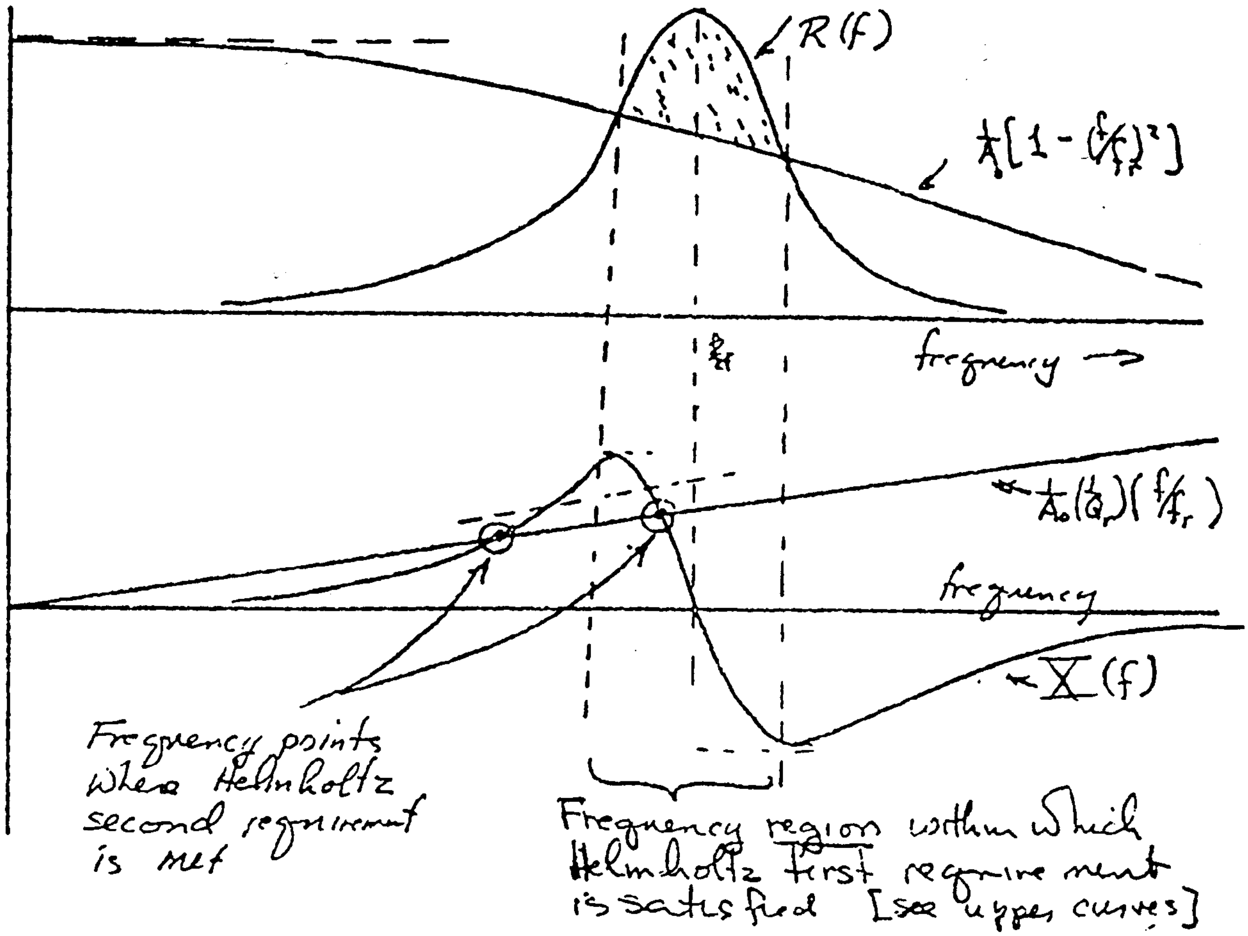
(C) Helmholtz's theory of excitation by reeds (no cooperations) says:

$R(f) \geq 1/A_o[1 - (f/f_r)^2]$ --The in-phase part of Z must equal or exceed the in-phase part of 1/A.

$X(f) = (1/A_o)(1/Q_r)(f/f_r)$ --The quadrature part of Z must equal the quadrature part of 1/A.

(D) Let us draw some diagrams to show the Helmholtz version because it will let us see perfectly well what is going on.

Consider the lowest frequency air column resonance talking to a reed well below f_r :



Only the higher-frequency of these is available to the oscillation.

$f_{playing}$ < freq of max Z

f_{player} < f_r ←HERE IS WHAT WE SEEK

<u>Increasing reed damping</u> lowers the playing frequency. Also (but not from today's work) <u>lowering</u> f_r lowers the playing frequency. ---Why?